Unavoidable Sets and Regularity of Languages Generated by (1,3)-Circular Splicing Systems

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Our aim

Focus on the still unknown relations between regular languages and CSSH systems.

Prove of necessary conditions for (1,3)-CSSH systems generating regular languages

Hybrid systems: relation between the regularity of the languages generated and the notion of unavoidability introduced by Ehrenfeucht, Haussler and Rozenberg in 1983.

Circular Splicing Systems and Open questions

Formal model of a generative mechanism of circular words inspired by a recombinant behaviour of circular DNA (Head 1992).

Defined by a finite alphabet A, an initial set I of circular words and a set R of rules.

Open questions:

- Decide whether a splicing language is regular
- Decide whether a regular circular language is a splicing language
- Characterize the structure of the regular splicing circular languages

Our Plan

- Background
- Outline of results
- Regular Sets and Unavoidable Sets
- Our approach: Hybrid Systems
- Future perspectives

The model: circular words and languages

Circular word

$^{\sim}w$ = equivalence class of w under *coniugacy relation* $\sim$

$$w, w' \in A^*, w \sim w' \iff w = xy, w' = yx$$

Example: $^{\sim}aaab = \{aaab, aaba, abaa, baaa\}$

Circular language

Circular language L = set of circular words.

$$\text{(linear) } X \rightarrow {}^{\sim}X = \{{}^{\sim}w \mid w \in X\} \text{ (circular)}$$

$$\text{(circular) } L \rightarrow Lin(L) = \{w \in A^* \mid {}^{\sim}w \in L\} \text{ (linear)}$$

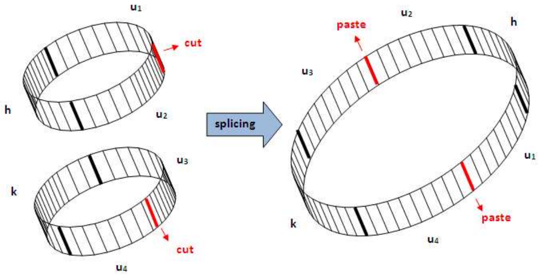
A circular language L is regular, i.e., $L \in Reg^{\sim}$, if and only if $Lin(L)$ is *regular*. (Similarly, $CF^{\sim}$, $CS^{\sim}$).

The model: circular splicing operation

Păun circular splicing operation

Let $r = u_1\#u_2\$u_3\#u_4 \in R$.

$(\sim w', \sim w'') \vdash_r \sim w$ se $w' = u_2 h u_1, w'' = u_4 k u_3, w = u_2 h u_1 u_4 k u_3$



Example:

$r = a\#b\$c\#d$ *and* $w' = bdda, w'' = daac, w = bddadaac.$

Then, $(\sim w', \sim w'') \vdash_r \sim w.$

The model: circular splicing systems

Păun Circular splicing system (1996)

Triple $S = (A, I, R)$:

- A finite alphabet
- I *initial* circular language ($I \subseteq \sim A^*$)
- R set of *rules* ($R \subseteq A^* \# A^* \$ A^* \# A^*$ con $\#, \$ \notin A$)
    - symmetric: $\forall u_1 \# u_2 \$ u_3 \# u_4 \in R$, $u_3 \# u_4 \$ u_1 \# u_2 \in R$.

Circular language generated by $S = (A, I, R)$

$L(S) =$ the smallest circular language that contains I and closed under iterated applications of the rules in R .

Finite Circular Splicing languages

C(Fin,Fin)= class of circular languages generated by finite circular splicing systems (I, R finite sets).

Results:

- Unlike the linear case, $C(Fin, Fin)$ is not comparable with $Reg^{\sim}$.
  Examples:
  $^{\sim}(aa)^* \in Reg^{\sim}$ *is a finite splicing circular language.*
  $^{\sim}a^n b^n \in CF^{\sim}$ *is a finite splicing circular language.*
- $C(Fin, Fin) \subset CS^{\sim}$ [Berstel, Boasson, Fagnot (2012)].
- Particular cases: *alphabetic, semi-simple (CSSH), marked, complete monotone* [see next slides].

Alphabetic systems and CSSH

Alphabetic systems [Berstel, Boasson, Fagnot (2012)]

$S = (A, I, R)$ is alphabetic if for each rule $u_1 \# u_2 \$ u_3 \# u_4 \in R$:

$$|u_i| \leq 1, 1 \leq i \leq 4$$

Semi-simple splicing systems (CSSH) [Ceterchi, Martín-Vide, Subramanian (2004)]

$S = (A, I, R)$ finite is a CSSH if for each $u_1 \# u_2 \$ u_3 \# u_4 \in R$:

$$u_1 u_2, u_3 u_4 \in A$$

If $u_1 u_2 = u_3 u_4 \in A$ then S is *simple*.

Known results

Alphabetic systems [Berstel, Boasson, Fagnot (2012)]

If I is context-free, then $L(S)$ is context-free.

Marked systems [De Felice, Fici, Zizza (2009)]

CSSH systems, $I = A = alph(R)$ and rules $a\#1\$b\#1$ ($(1,3)$ type).

- characterization of the structure of $L(S)$
- decidable to estabilish whether $L(S)$ is regular

Complete monotone systems [Bonizzoni, De Felice, Zizza (2010)]

CSSH systems, $alph(I) = A$, $R = \{a\#1\$b\#1 | a, b \in A\} = A \times A$.

- characterization of the systems that generate regular splicing circular languages
- decidable to estabilish whether $L(S)$ is regular

Regular Languages and Unavoidable Sets

Unavoidable Set [Ehrenfeucht, Haussler, Rozenberg (1983)]

Let $X, Y \subseteq A^*$. Y is *unavoidable* in X if $\exists k_0 \in N$ such that $\forall x \in X$, with $|x| > k_0$, $\exists y \in Y$ such that $x = x_1 y x_2$.

Examples:
$Y = \{aa, b\}$ is unavoidable in A^*.
$Y = \{aa, ab\}$ is avoidable in A^*. In fact, Y is avoidable in b^*.

- **A complete $(1,3)$-CSSH system generates a regular circular language if and only if $Lin(I)$ is unavoidable [Bonizzoni, De Felice, Zizza (2010)].**

Generalization of a result in [Ehrenfeucht, Haussler, Rozenberg (1983)]

Let $S = (A, I, R)$ be a $(1,3)$-CSSH system. If $L(S)$ is regular then $Lin(I)$ is unavoidable in $Pref(Lin(L(S)))$.

Hybrid Systems

Definition (Hybrid System)

(1, 3)-CSSH system $S = (A, I, R)$ such that:

- $A = alph(R) = alph(I)$
- For any $a_i, a_j \in A$ there are $b_1, \ldots, b_k$, with $k \geq 2$ such that: $b_1 = a_i$, $b_k = a_j$ and for all $h \in \{1, \ldots, k-1\}$, $(b_h, b_{h+1}) \in R$.

Examples:

- *The* $(1, 3)$*-CSSH system* $S = (A, I, R)$*, with* $A = \{a, b\}$*,* $I = {\sim}\{ab, a, b\}$ *and* $R = \{(a, b)\}$*, is hybrid.*
- *The* $(1, 3)$*-CSSH system* $S' = (A, I', R')$*, with* $I = {\sim}\{ab, a\}$ *and* $R' = \{(a, a)\}$*, is not hybrid.*

Hybrid Systems

Proposition

If $S = (A, I, R)$ is a hybrid system, then $Pref(Lin(S)) = A^*$.

Corollary (1)

Let $S = (A, I, R)$ be a hybrid system, let $Y = Lin(I)$. If $L(S)$ is a regular circular language, then Y is unavoidable.

Corollary (2)

Let $S = (A, I, R)$ be a hybrid system. If $L(S)$ is a regular circular language, then for any $a \in A$ there exists $n \in N$ such that $a^n \in I$.

A necessary condition for Regularity

Proposition

Let $S = (A, I, R)$ be a $(1, 3)$-CSSH, let $a \in A$. If $(a, b) \in R$ for all $b \in A \setminus \{a\}$ and $Lin(I) \cap a^* = \emptyset$, then $L(S)$ is not regular.

Future Perspectives

Conjecture

Let $S = (A, I, R)$ be a hybrid system, let $Y = Lin(I)$. $L(S)$ is regular if and only if $d(R) < 4$ and Y is unavoidable, where $d(R)$ is the diameter of R.

Example 1:
The $S = (A, I, R)$ *with* $A = \{a, b\}$, $I = {}^{\sim}\{ab, a, b\}$ *and* $R = \{(a, b)\}$.

- $d(R) < 4$
- $Lin(I)$ *is unavoidable.*

$$Lin(L(S)) = \{a, b\}^+ \setminus (a^+a \cup b^+b) \in Reg$$

Example 2:
The $S = (A, I, R)$ *with* $A = \{a, b\}$, $I = {}^{\sim}\{ab\}$ *and* $R = \{(a, b)\}$.
Observe that $d(R) < 4$ *but* $Lin(I)$ *is avoidable.*

$$Lin(L(S)) \cap a^* b^* = \{a^n b^n | n > 0\} \notin Reg$$