Are Good-for-games Automata Good for Probabilistic Model Checking?

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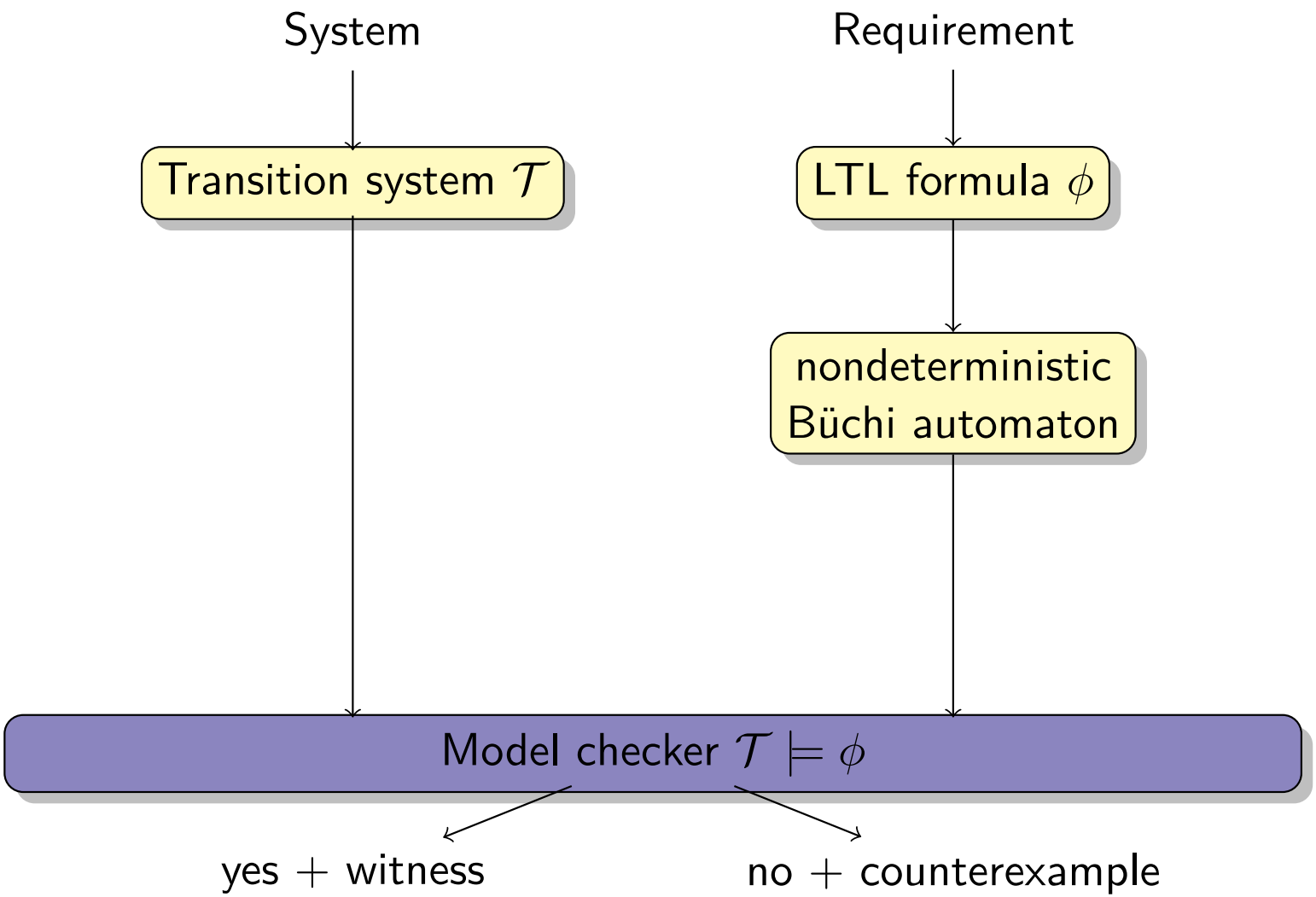
March 15, 2014

LTL model checking



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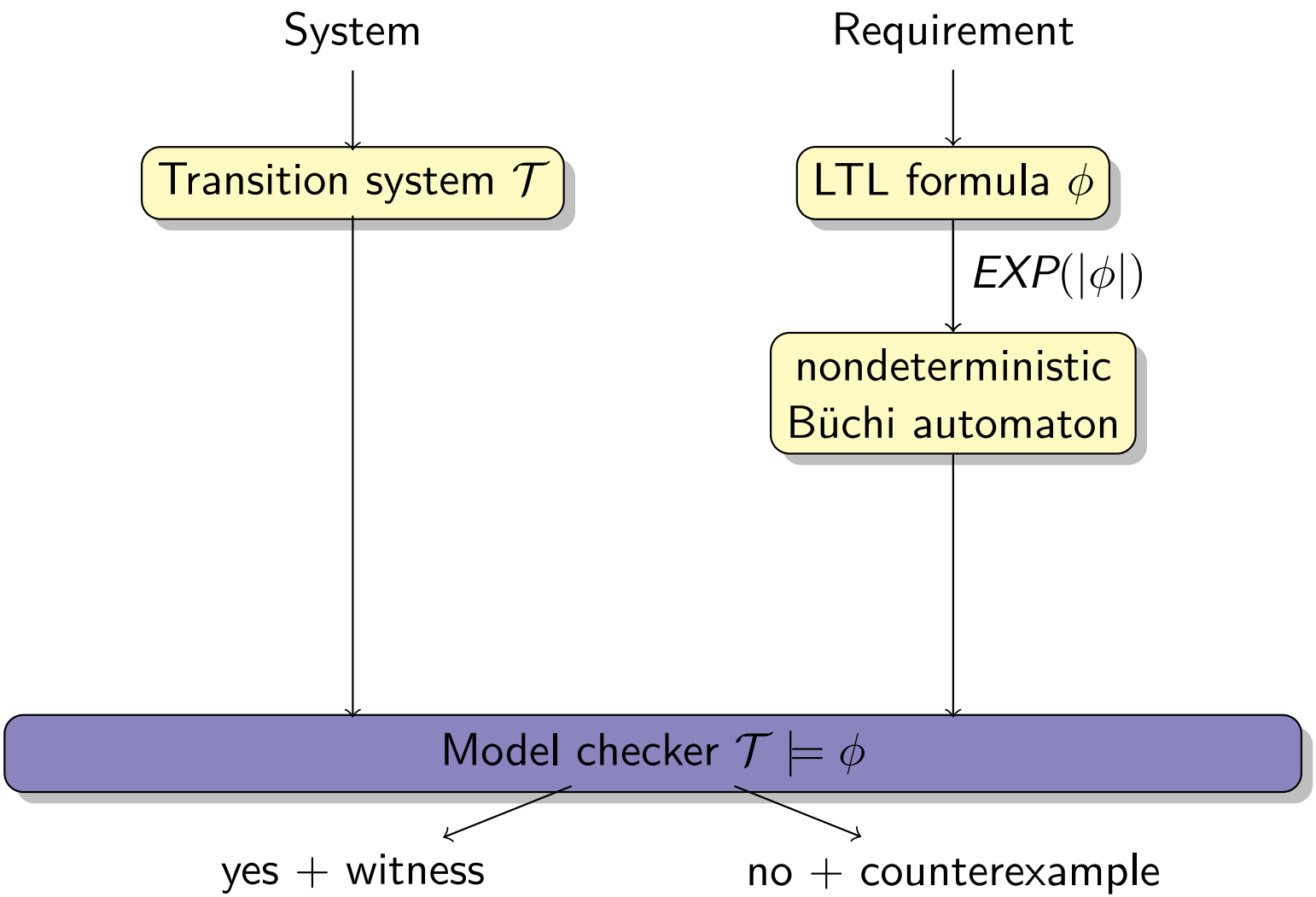
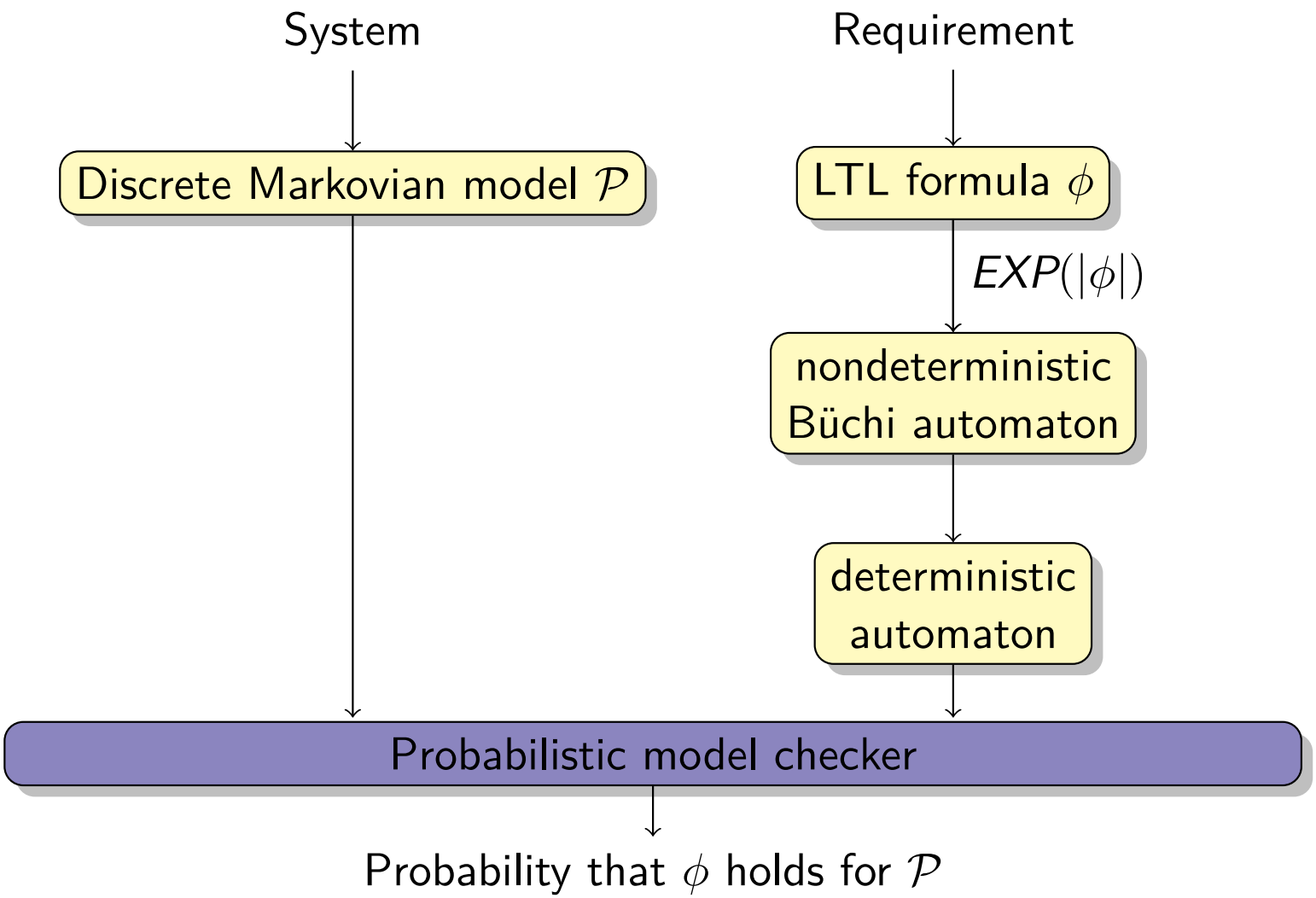


Probabilistic model checking



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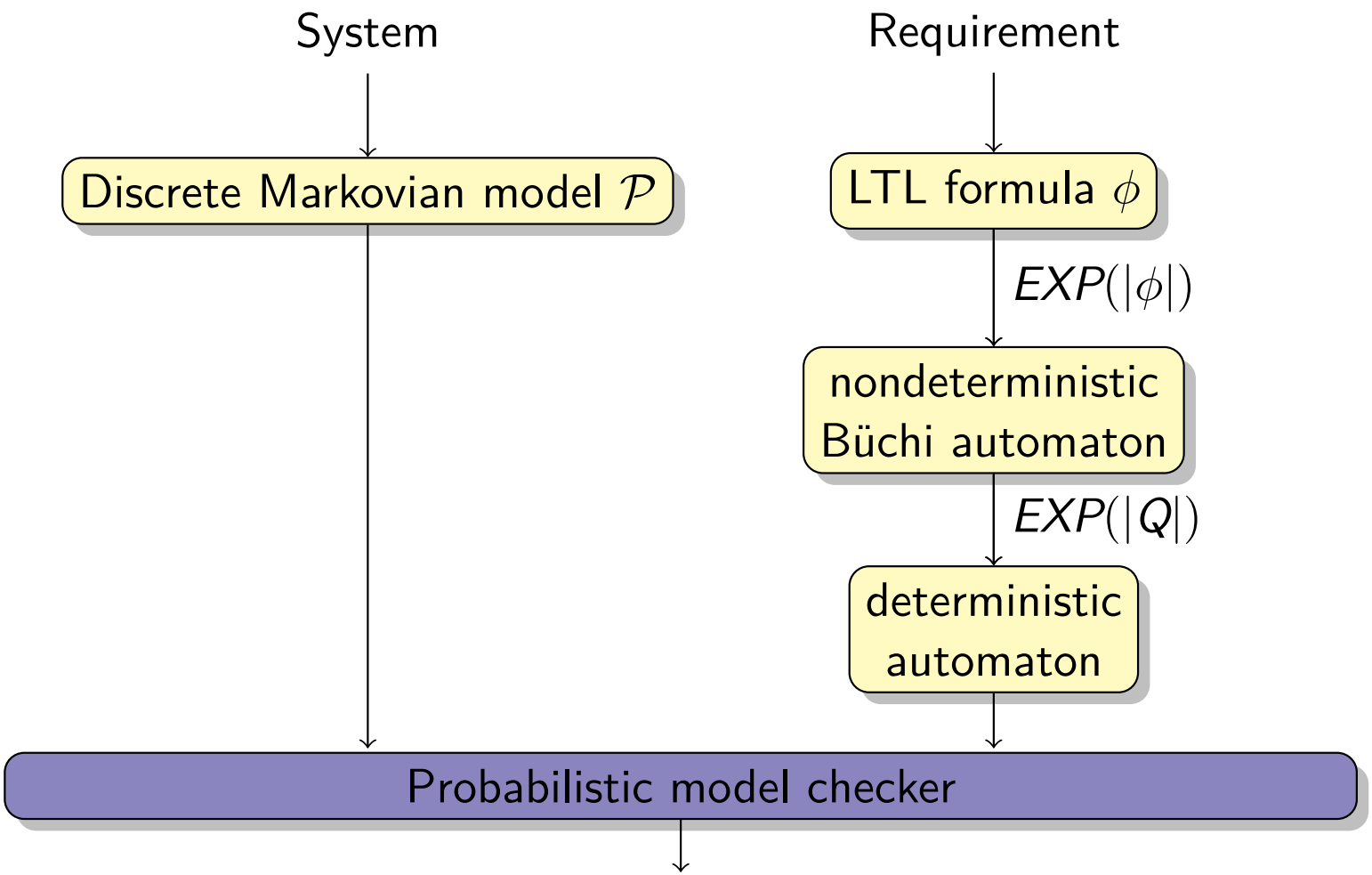

Lower bound NBA $\rightarrow$ det. automaton by Michel, 1988

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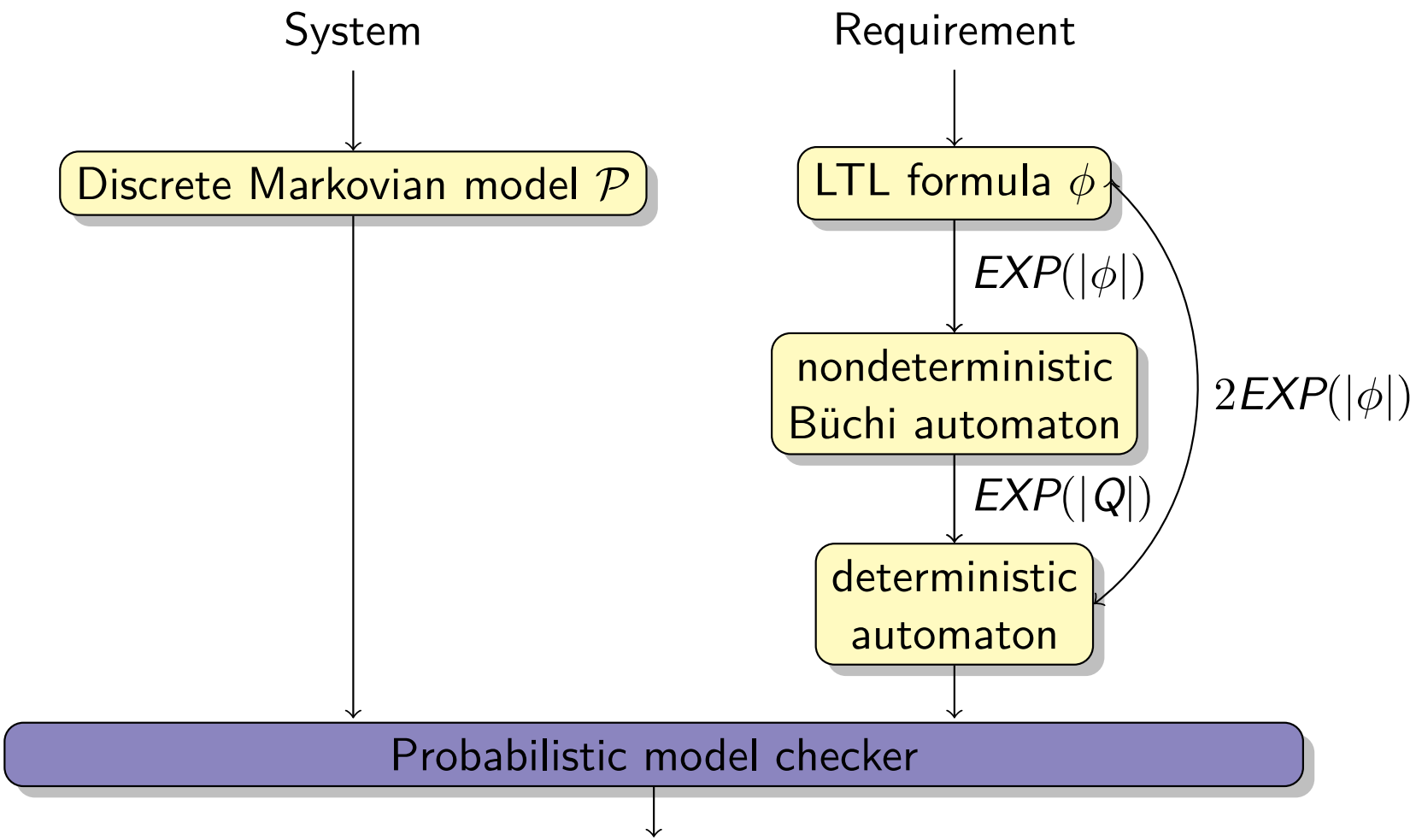

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Lower bound LTL $\rightarrow$ det. automaton by Kupferman, Rosenberg, 2011

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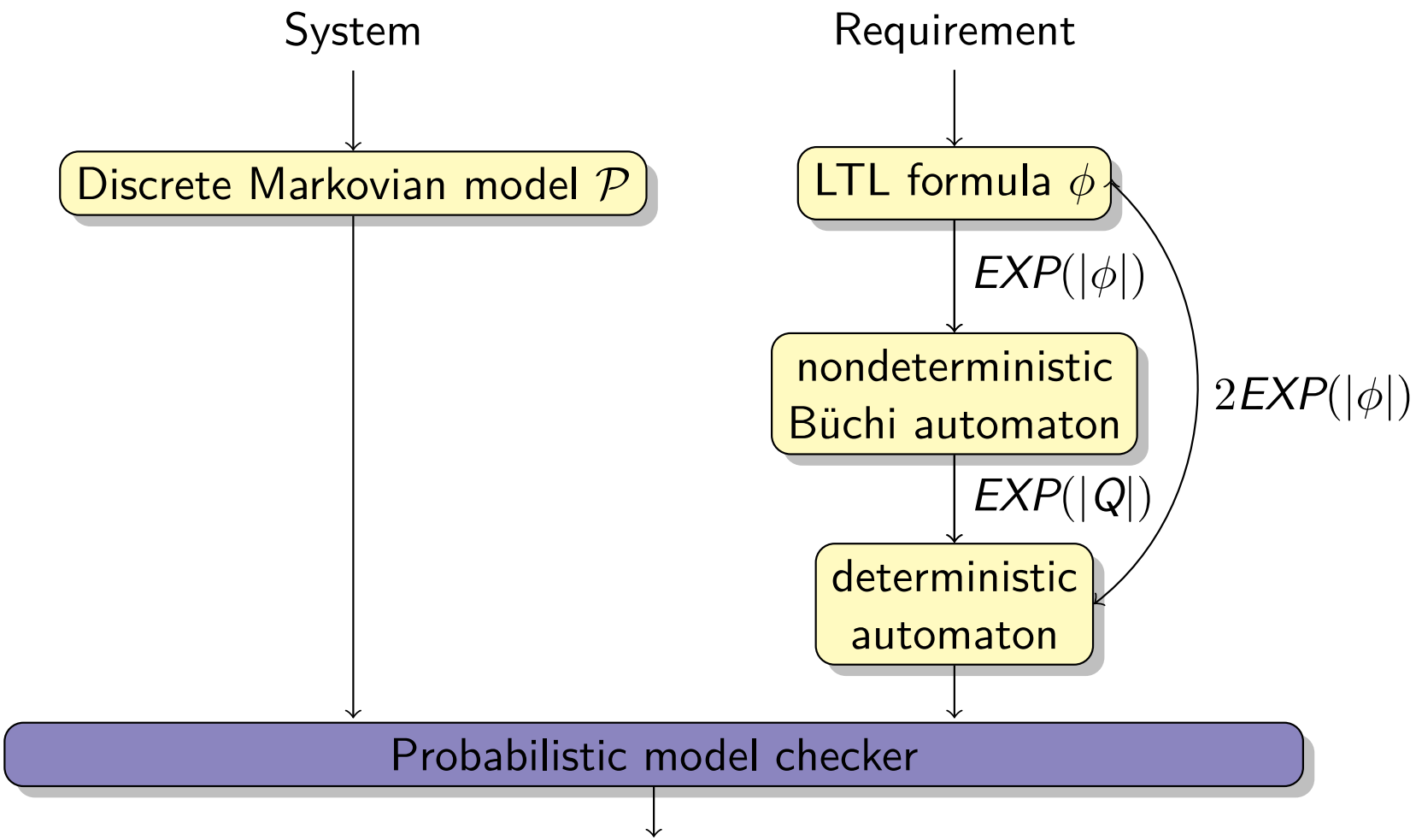

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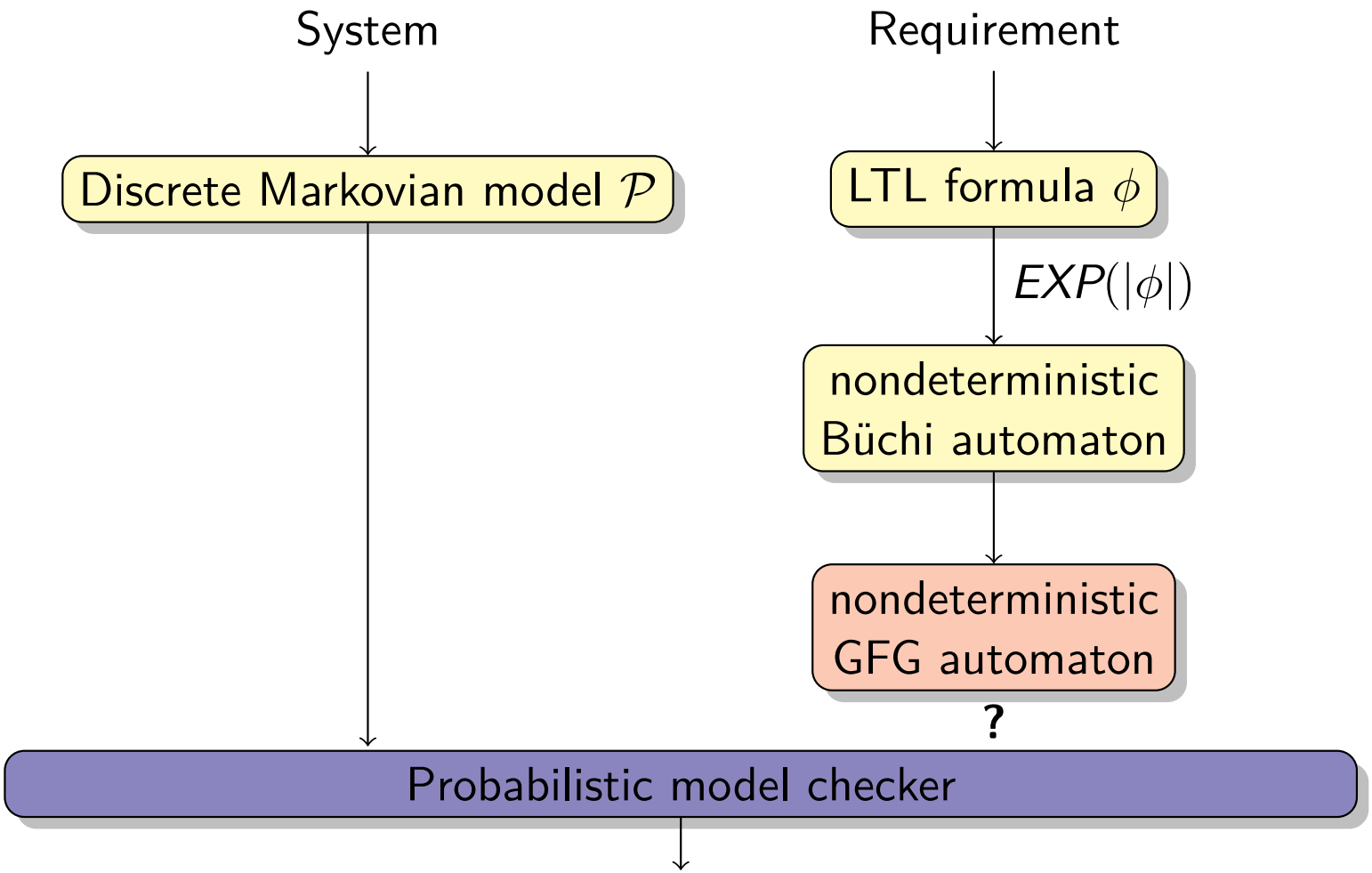

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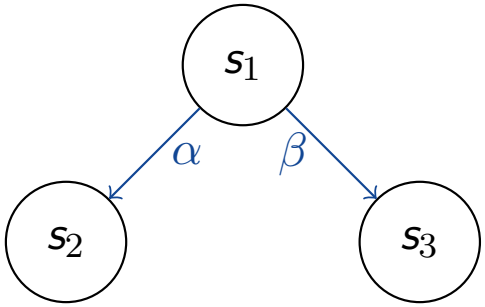
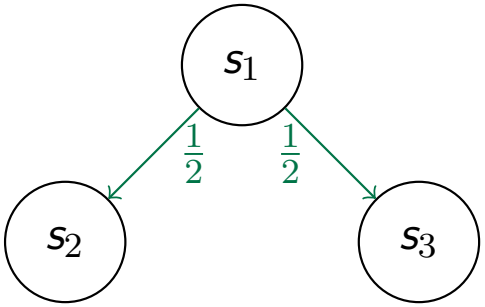
MDP: Probabilism and nondeterminism



Transition system:
Nondeterminism



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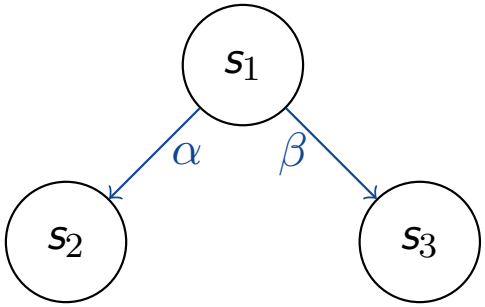

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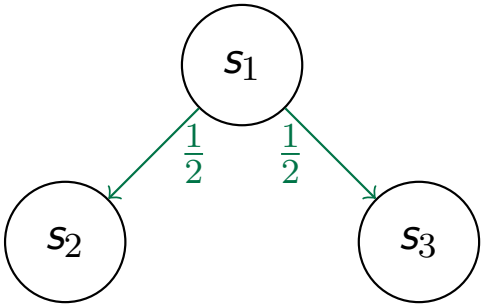

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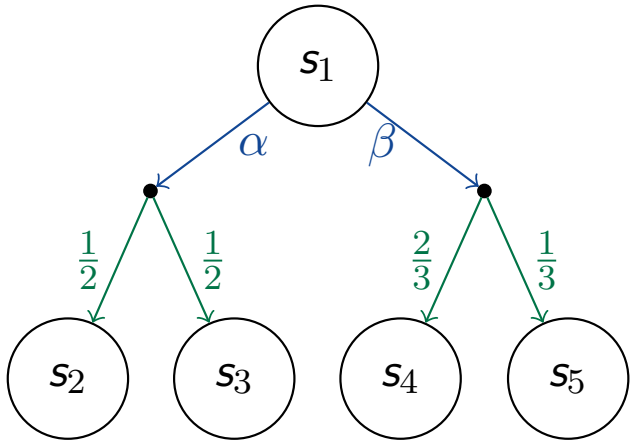

Markov decision process: Nondeterminism and probabilism

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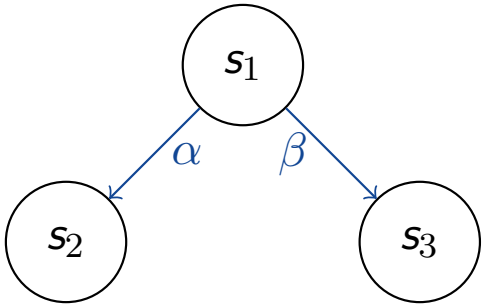

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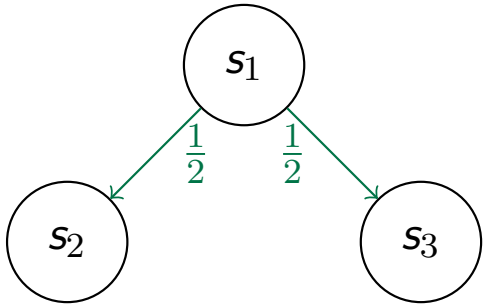

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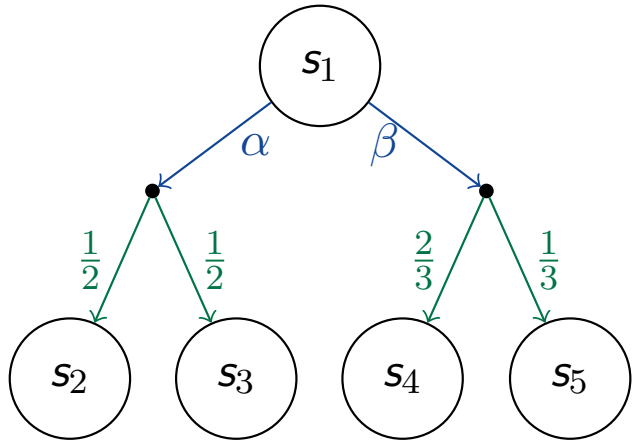

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Scheduler $\mathfrak{S} : S^+ \to Act$ chooses for given history of states next action

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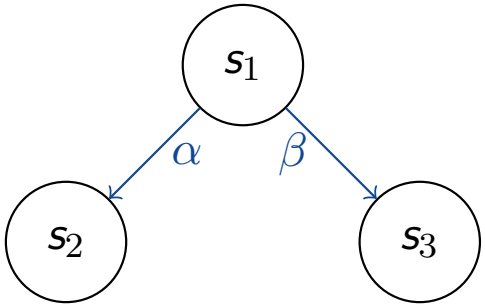

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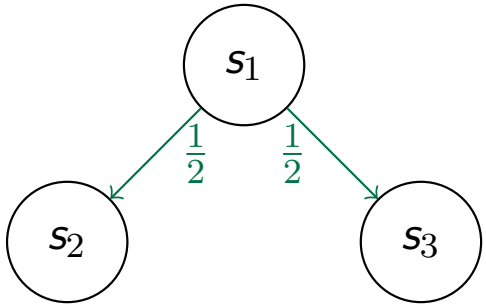

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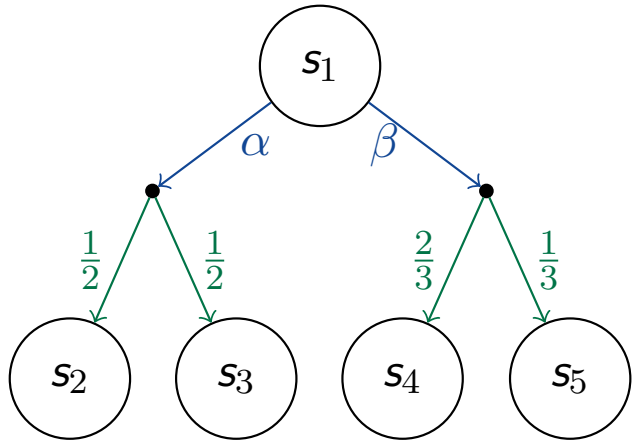

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$Pr_{\mathcal{P}}(\Diamond s_4)$?

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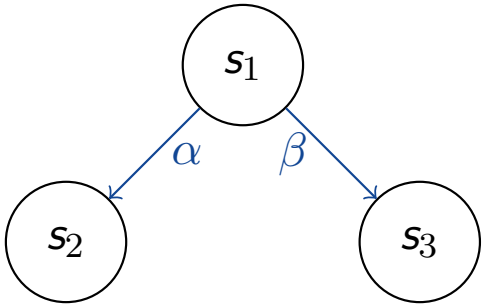

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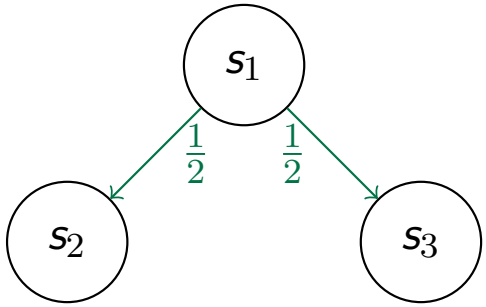

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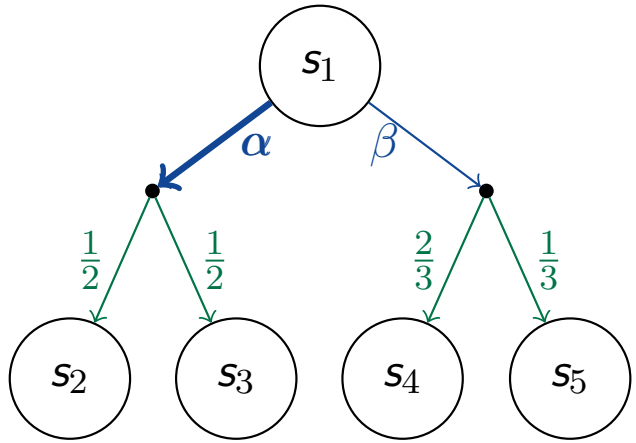

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$Pr_{\mathcal{P}}(\Diamond s_4)$? $\longrightarrow$ $Pr^{min}_{\mathcal{P}}(\Diamond s_4) = 0$

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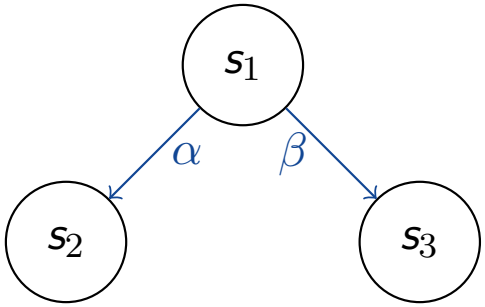

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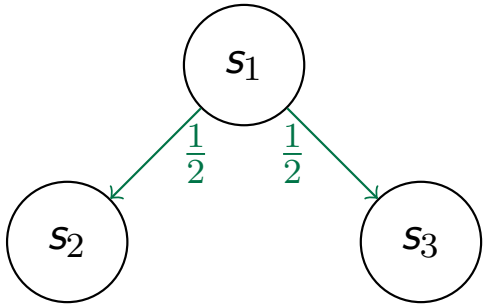

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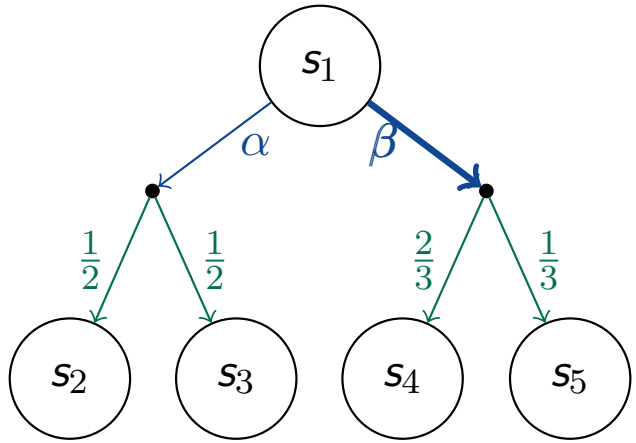

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$$Pr_{\mathcal{P}}(\Diamond s_4)\ ? \begin{cases} \to Pr^{max}_{\mathcal{P}}(\Diamond s_4) = \frac{2}{3} \\ \to Pr^{min}_{\mathcal{P}}(\Diamond s_4) = 0 \end{cases}$$

Good-for-games automata

- Property of nondeterministic automata

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- Used to avoid determinisation for LTL synthesis

Good-for-games automata

deterministic automata

$w \in \mathcal{L}(\mathcal{A}) \Leftrightarrow$ *unique run is an accepting run*

Nondeterministic automata

$w \in \mathcal{L}(\mathcal{A}) \Leftrightarrow$ *exists an accepting run*

Good-for-games automata

deterministic automata
$w \in \mathcal{L}(\mathcal{A}) \Leftrightarrow$ *unique run is an accepting run*

Good-for-games
exists a strategy s.t.
for all $w \in \mathcal{L}(\mathcal{A})$ *strategy-induced run accepting*

Nondeterministic automata
$w \in \mathcal{L}(\mathcal{A}) \Leftrightarrow$ *exists an accepting run*

Good-for-games

- two player game on the automaton:

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An ω-automaton is *good-for-games* if there exists a strategy, s.t. for every word $w \in \mathcal{L}(\mathcal{A})$ the induced run is accepting.

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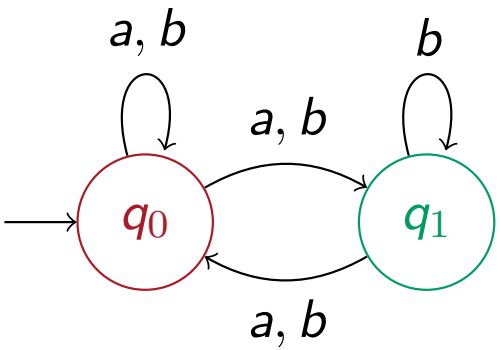
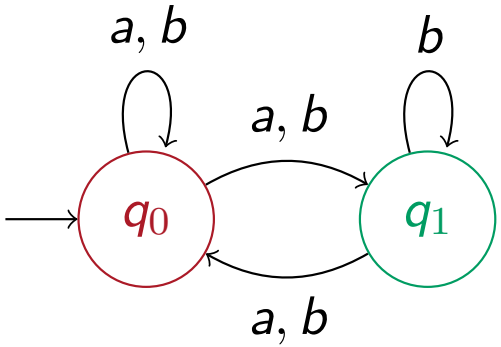


$$Acc = \Diamond\Box\neg q_0 \wedge \Box\Diamond q_1$$

Run π is accepting $\Leftrightarrow q_0 \notin \mathit{inf}(\pi) \wedge q_1 \in \mathit{inf}(\pi)$

$$\mathcal{L}(\mathcal{A}) = \mathcal{L}((a+b)^* b^\omega)$$

Strategy



GFG strategy

$$D(\pi) = \begin{cases} q_0 & \text{if } \pi \text{ ends with } q_1 a \\ q_1 & \text{otherwise} \end{cases}$$

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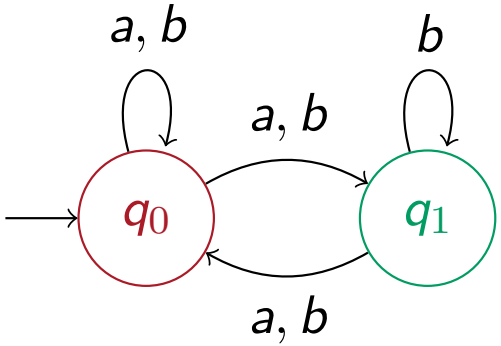

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word	a
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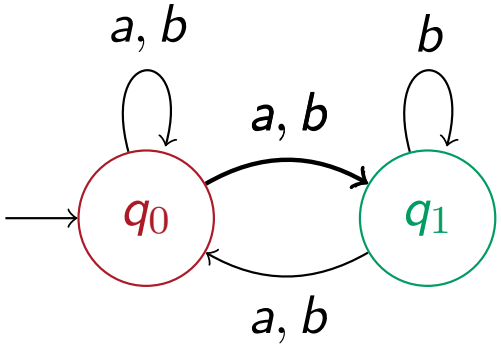

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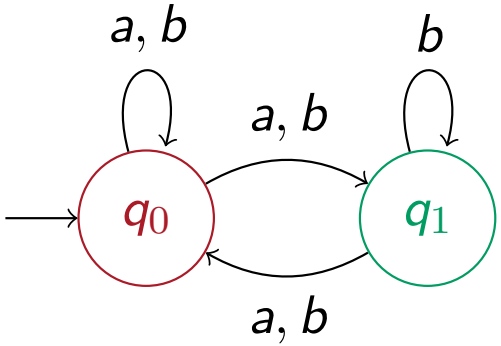

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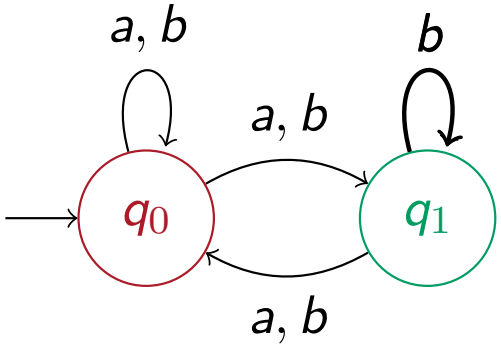
play	$q_0 a$	$q_1 b$

Strategy



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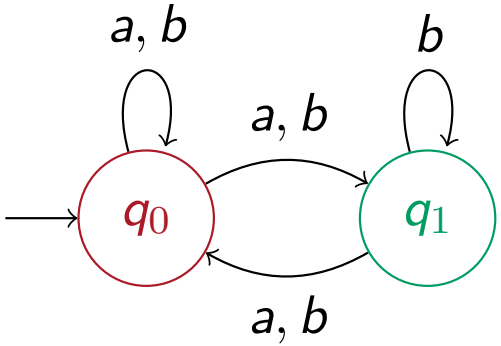

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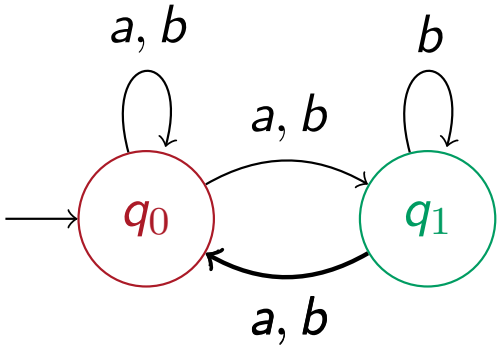

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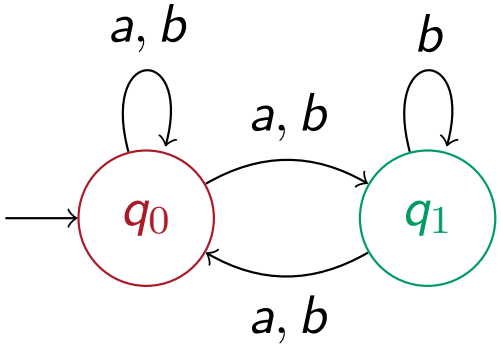

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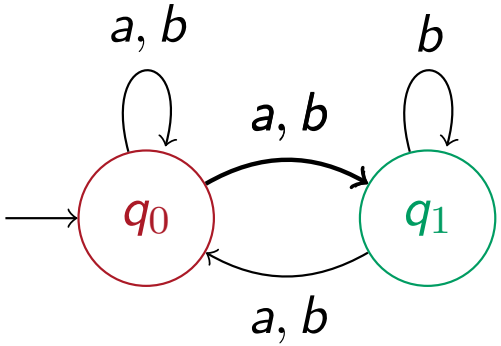

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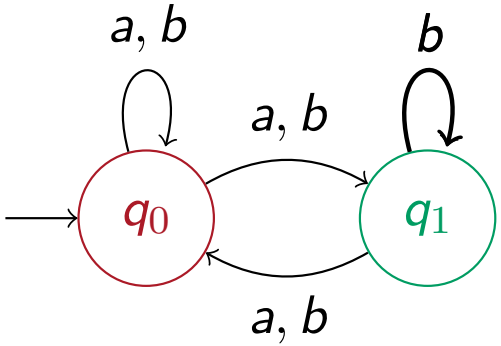

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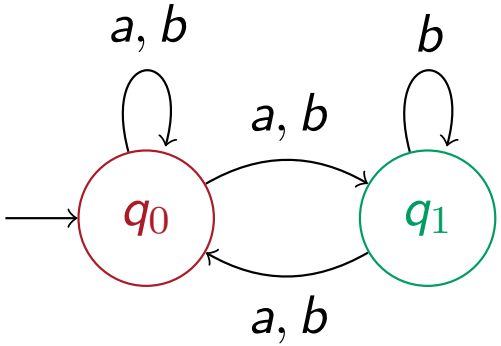

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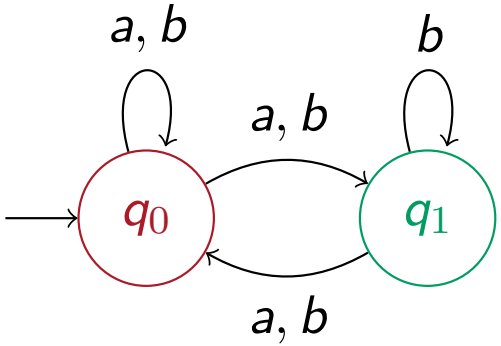

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- For an accepted word player 1 has to choose b from some point on
- Every induced D-run remains in q_1 from some point on

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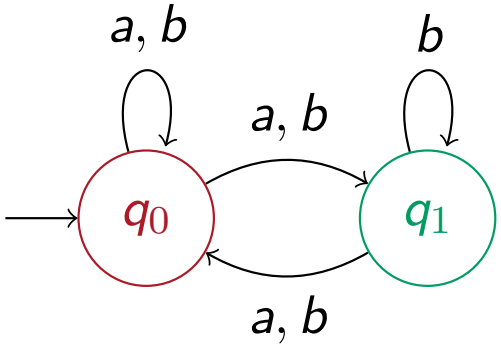

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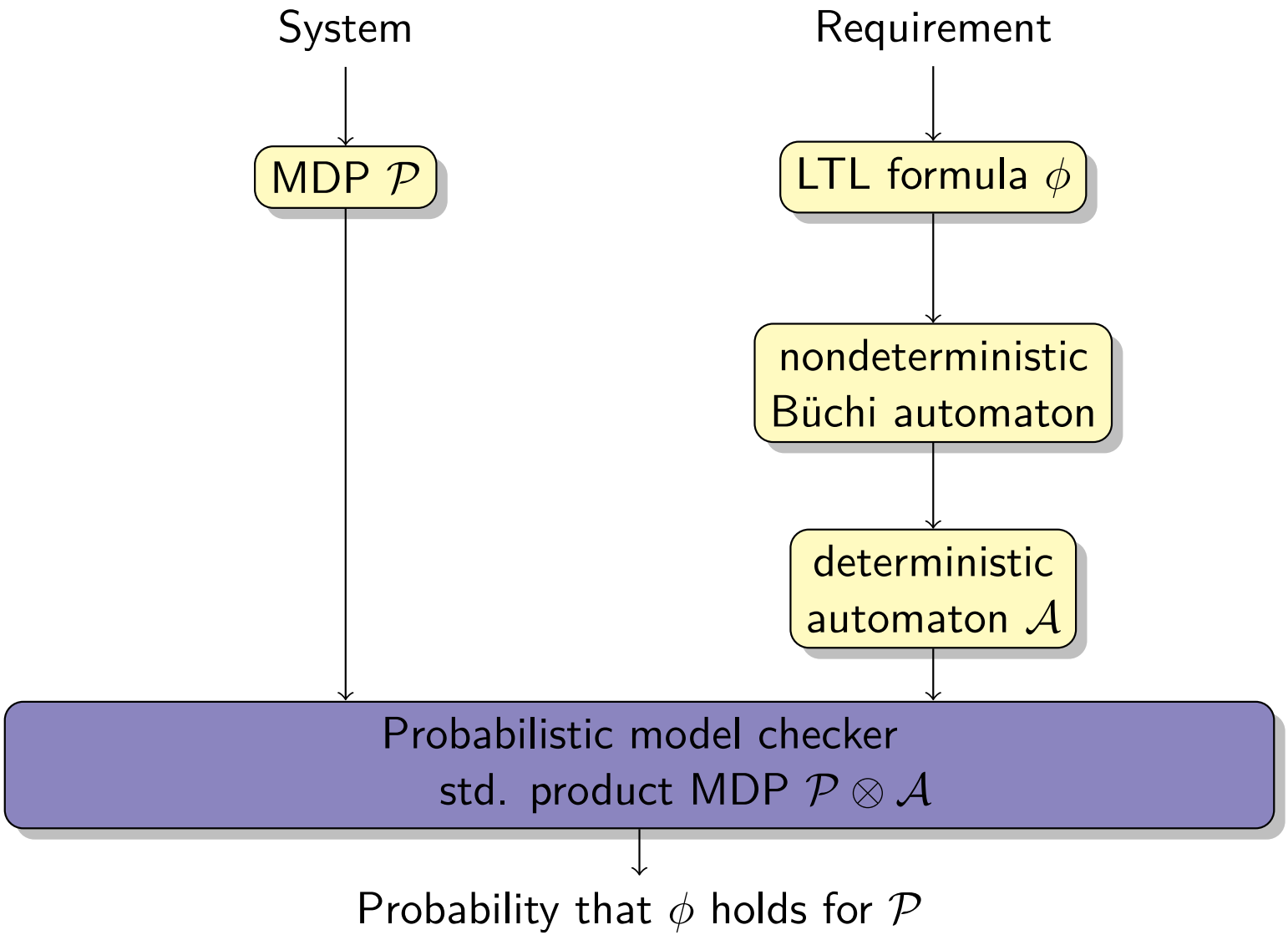
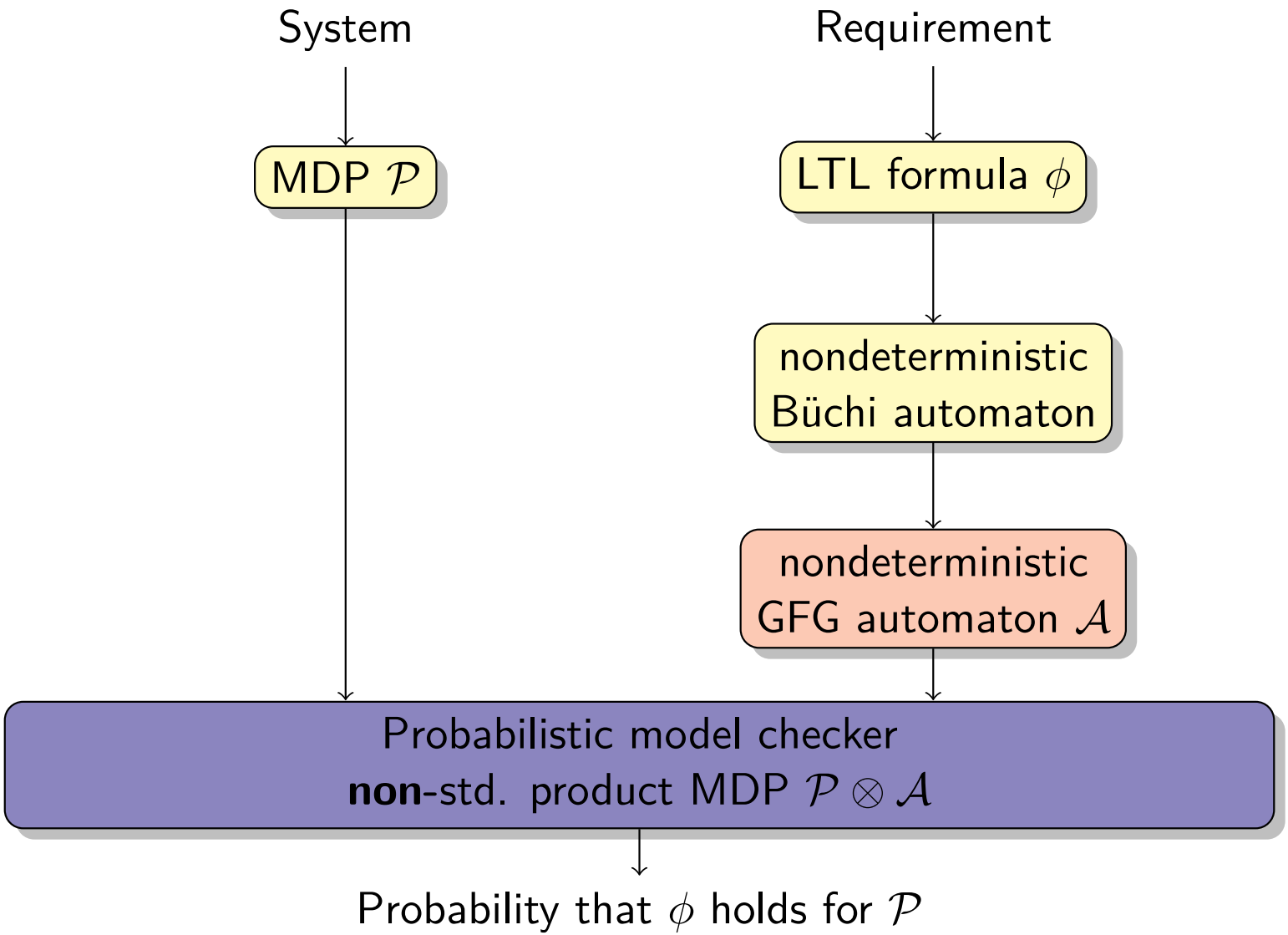
word	a	b	a	b^ω	
play	$q_0 a$	$q_1 b$	$q_1 a$	$q_0 (b$	$q_1)^\omega$

- For an accepted word player 1 has to choose b from some point on
- Every induced D-run remains in q_1 from some point on
- Every induced D-run is accepting

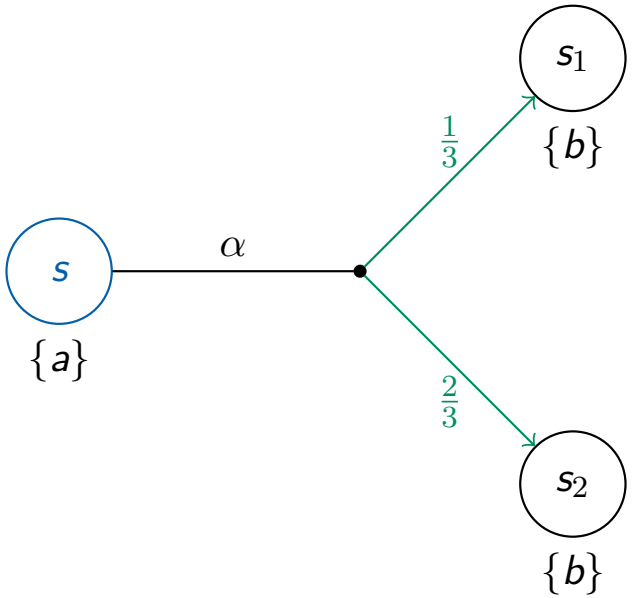
Probabilistic model checking

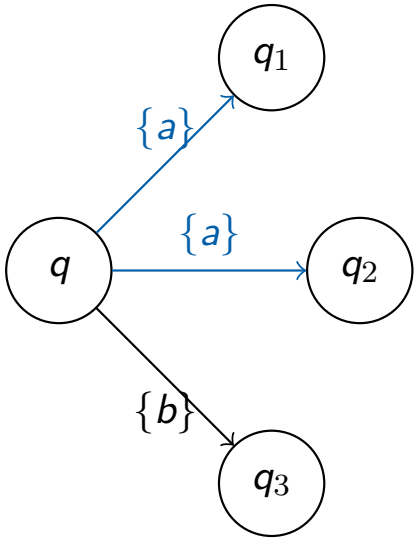


Probabilistic model checking

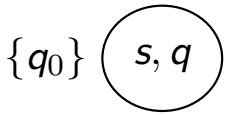


Product MDP: $\mathcal{P} \otimes \mathcal{A}$





For state (s, q) in $\mathcal{P} \otimes \mathcal{A}$:



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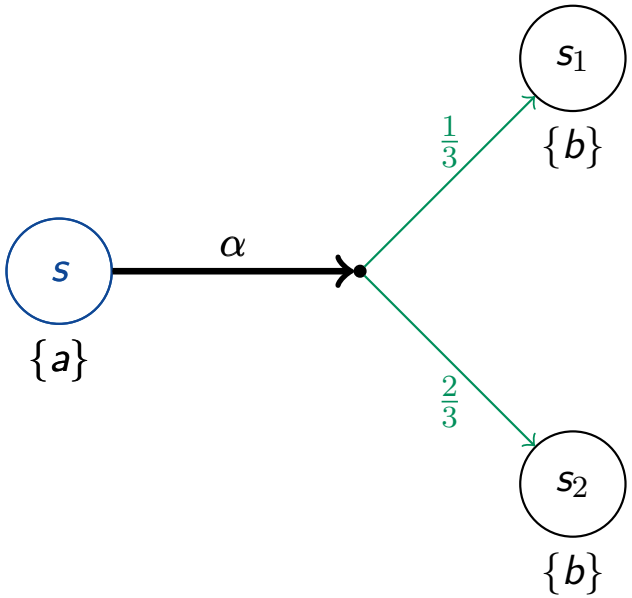

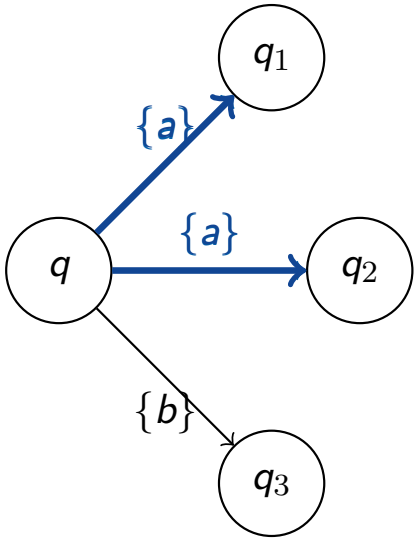

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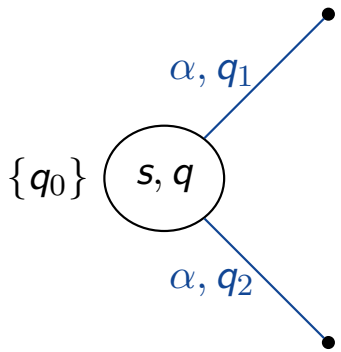
- enabled actions: (α, q') for $\alpha \in Act(s)$ and $q' \in \delta(q, L(s))$



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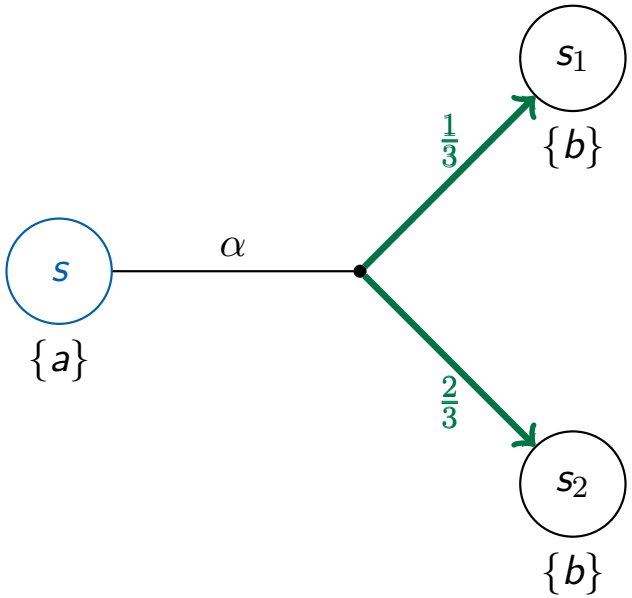

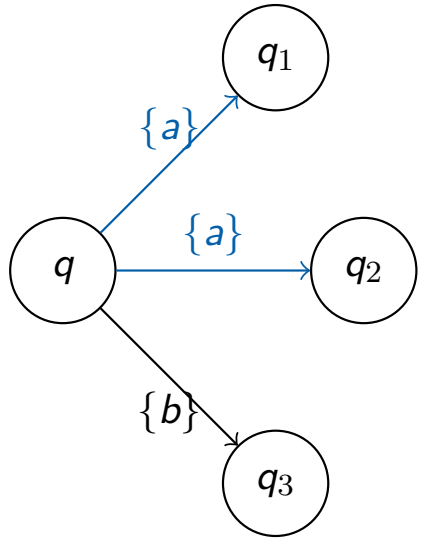

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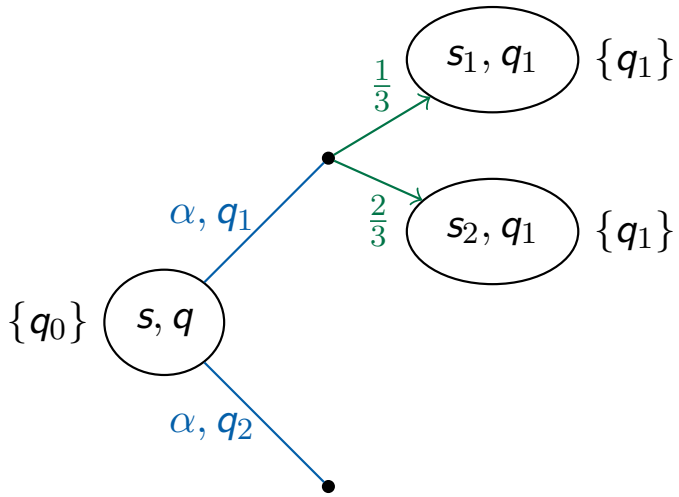

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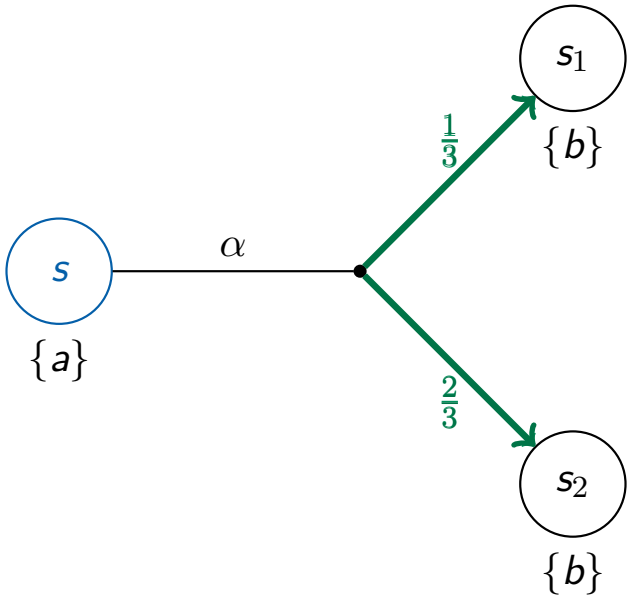

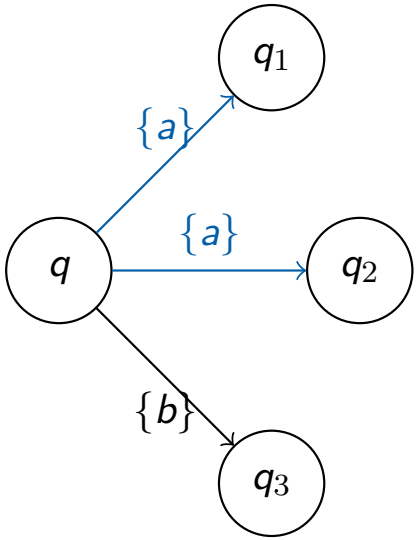

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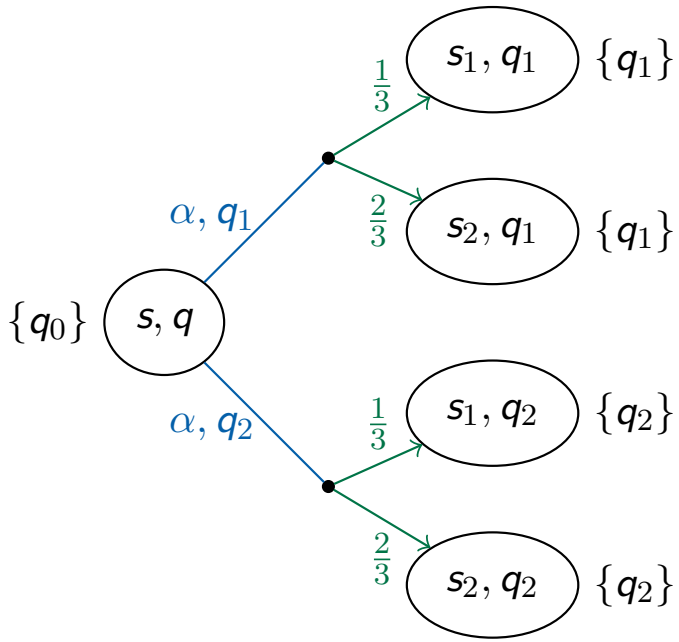

MDP analysis with GFG

Theorem

Let $\mathcal{P}$ be an MDP and $\mathcal{A}$ be an GFG ω-automaton. Then

$$Pr^{max}_{\mathcal{P}\otimes\mathcal{A}}(\langle s_0, q_0\rangle \models Acc) = Pr^{max}_{\mathcal{P}}(s_0 \models \mathcal{L}(\mathcal{A}))$$

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Proof idea of "$\geq$":

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Proof of "$\leq$" is rather technical

From LTL to GFG

- NBA to GFG nondet. parity automaton with at most $2^n \cdot n^{2n}$ states from Henzinger and Piterman, 2006

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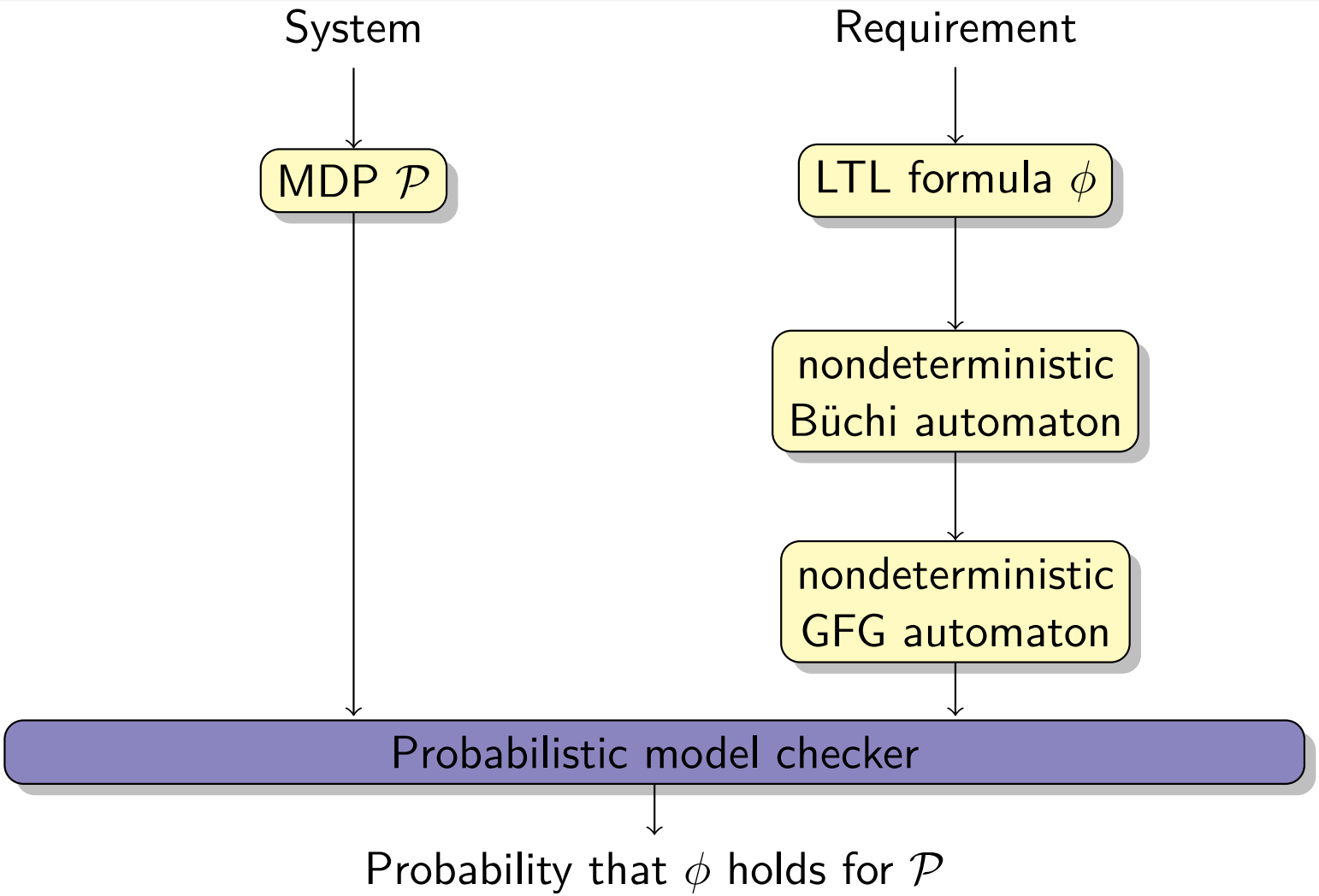
- intended for symbolic implementation

Probabilistic model checking



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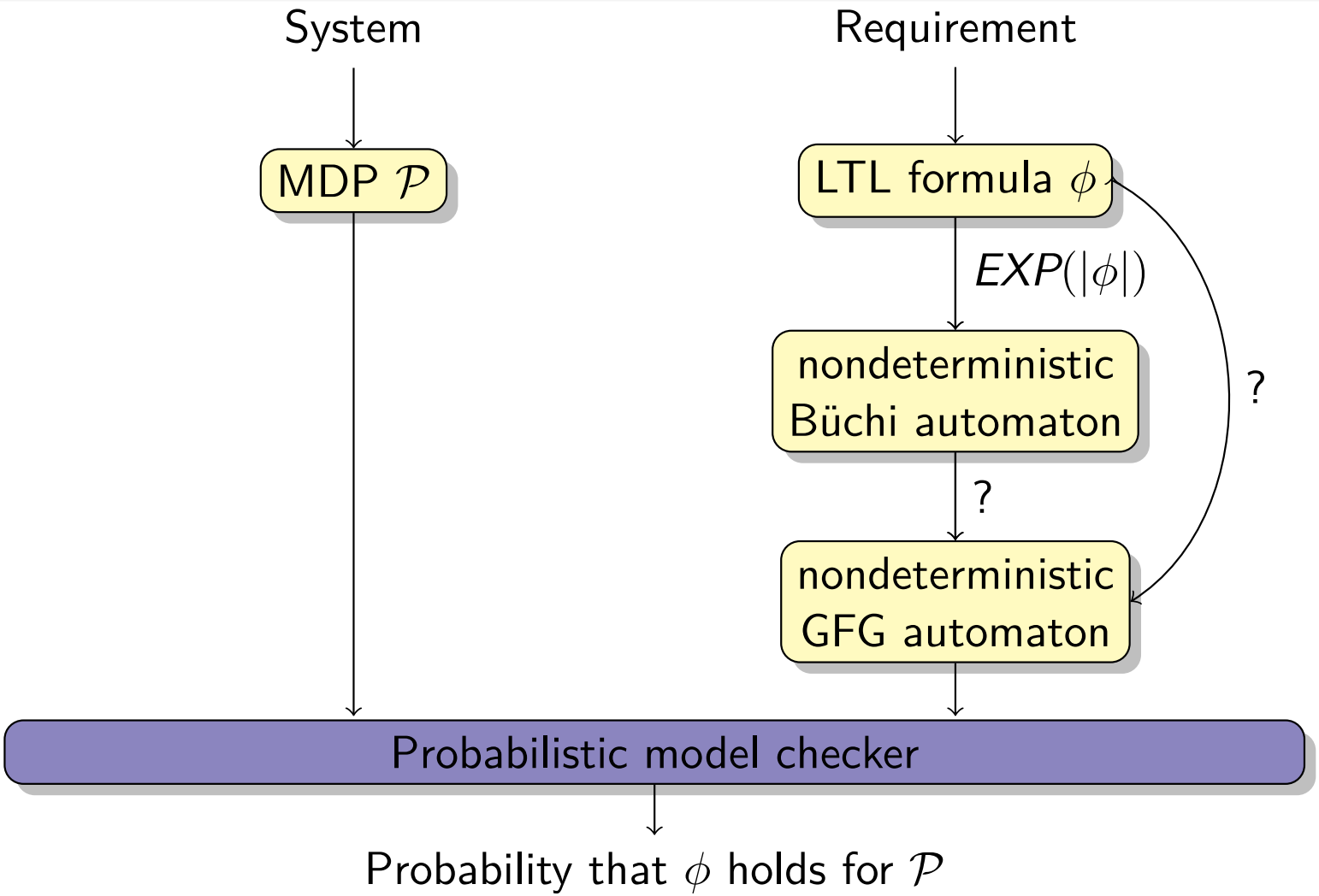

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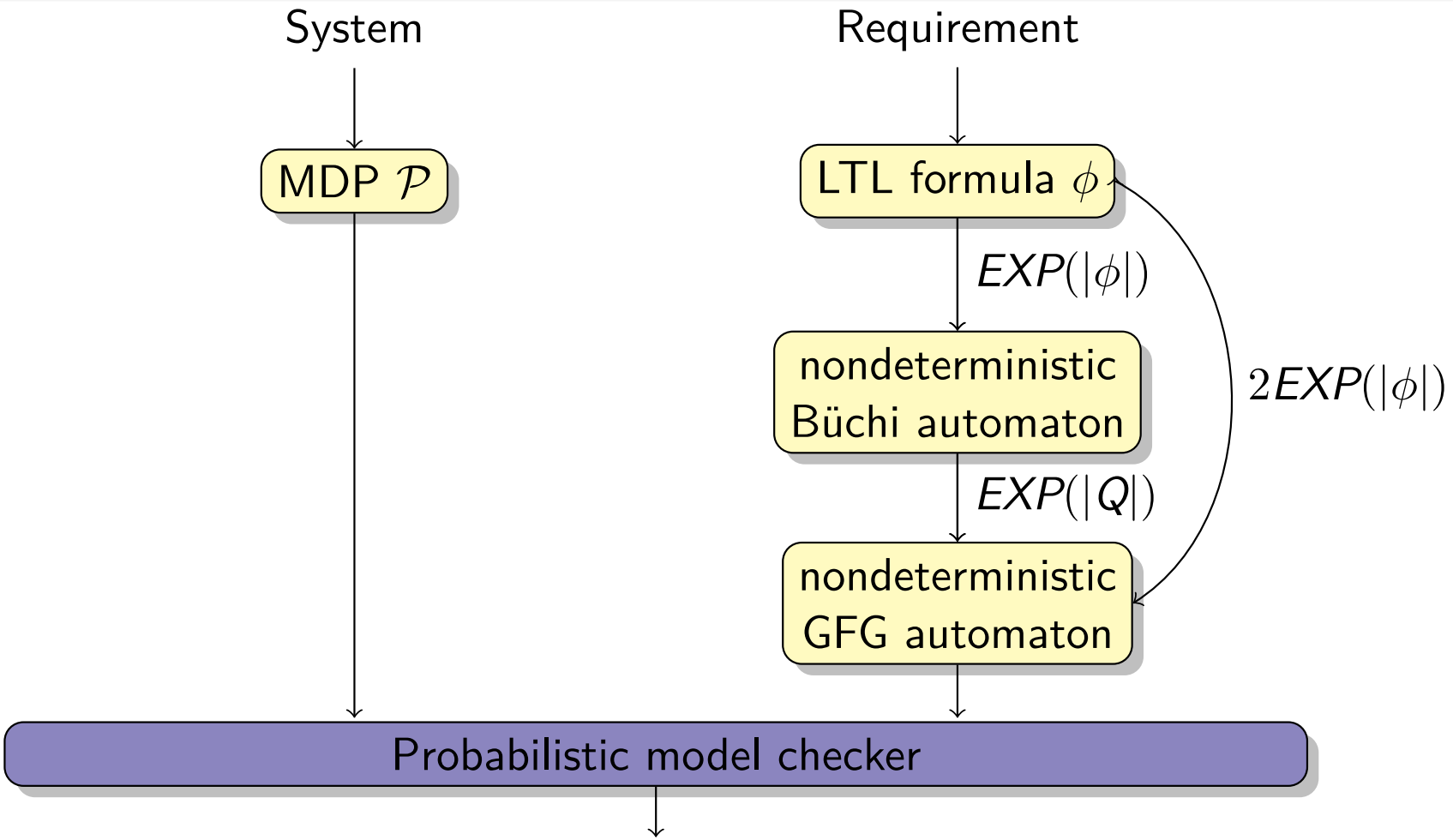
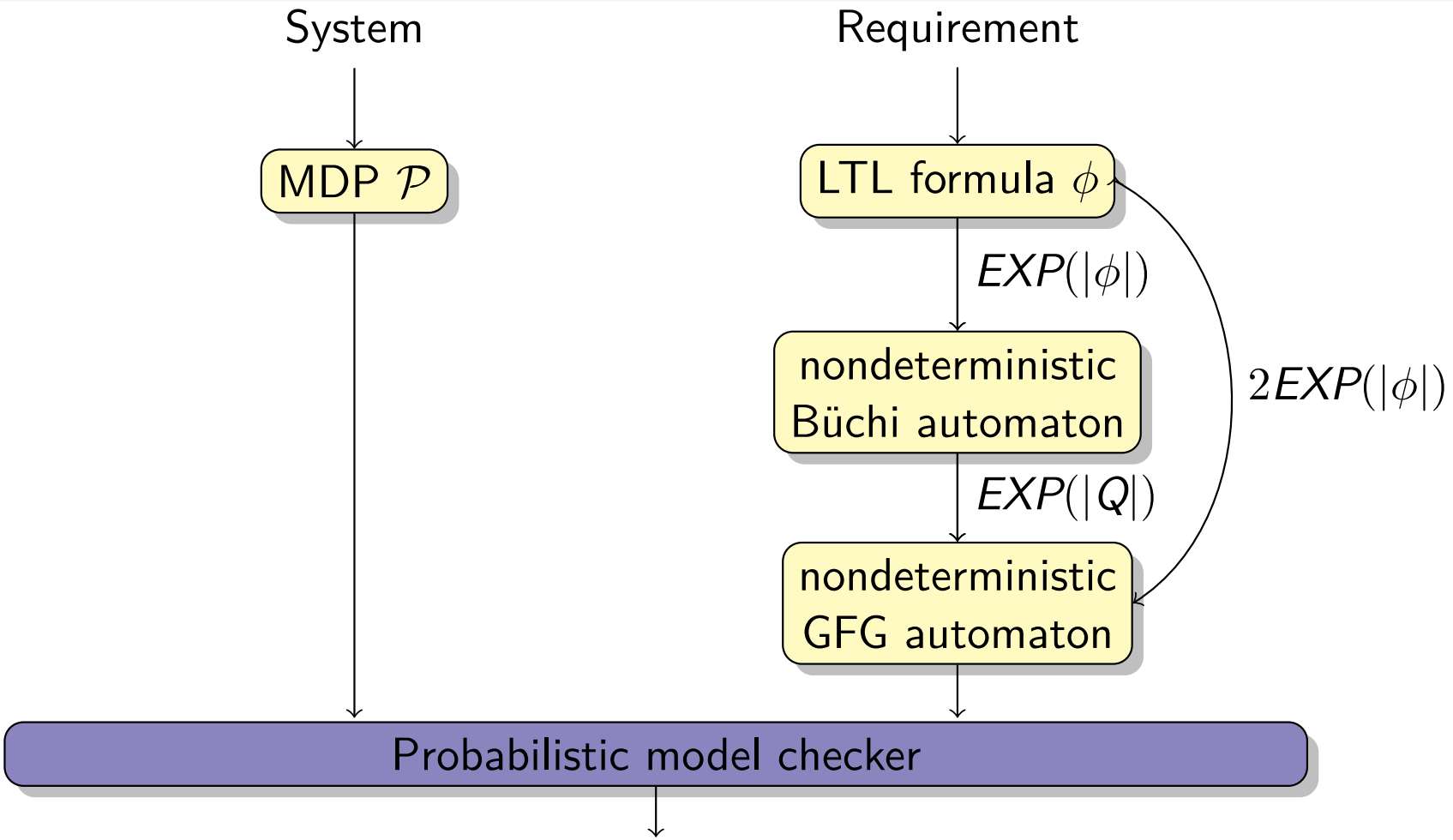


Lower bound NBA $\rightarrow$ GFG by Henzinger,Piterman, 2006

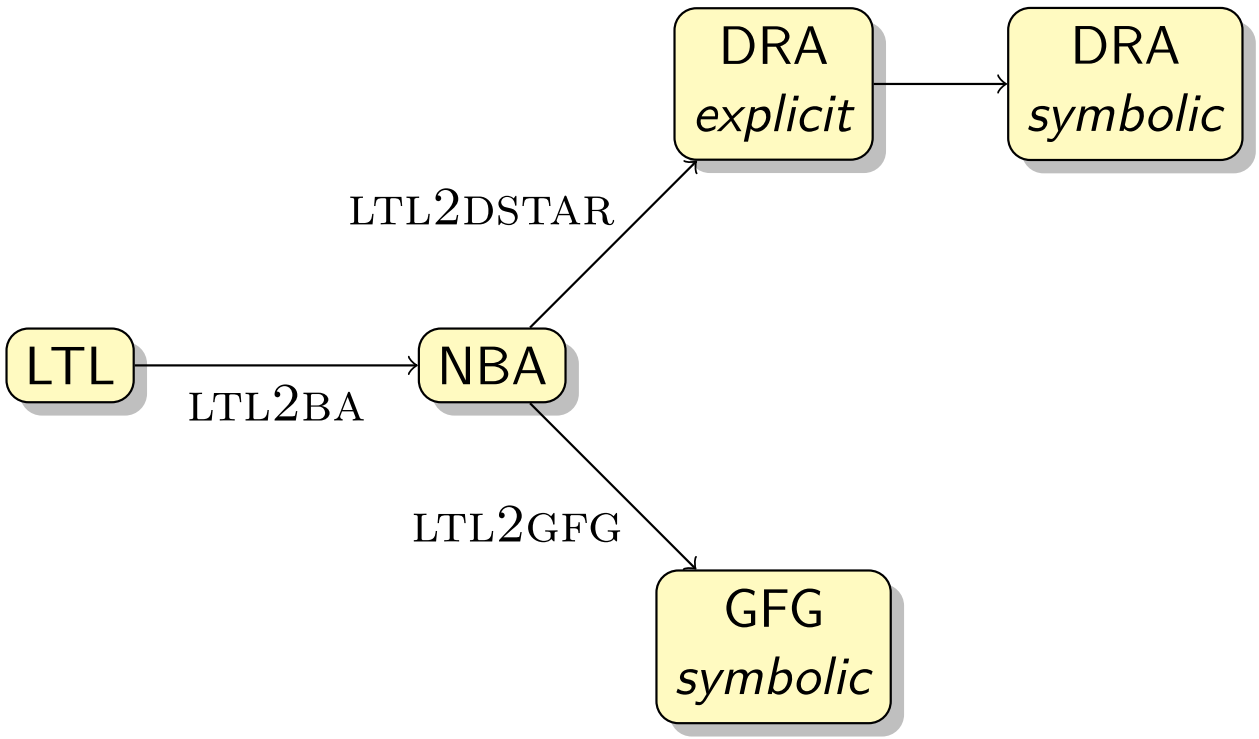
Probabilistic model checking



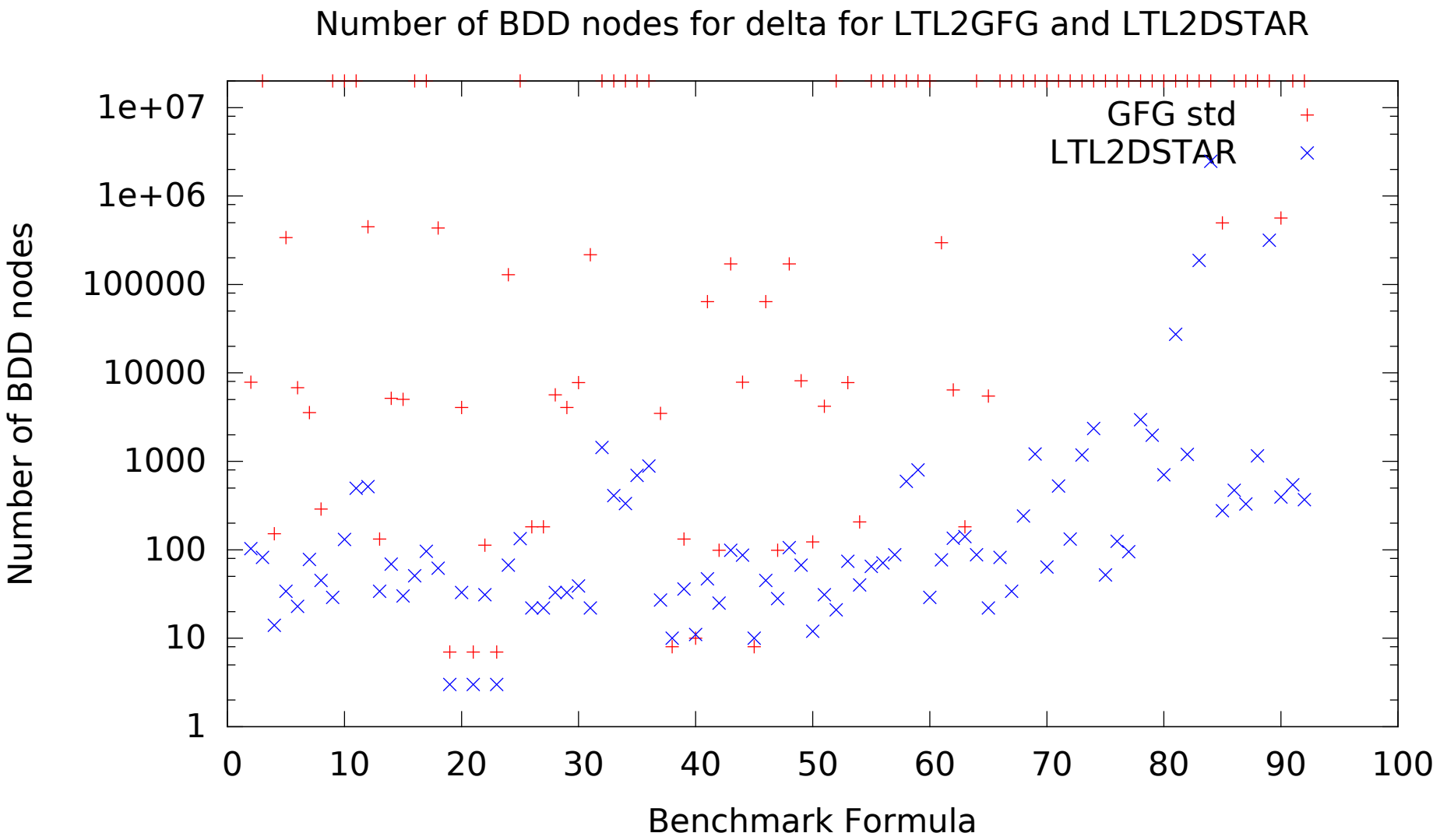
Lower bound NBA $\rightarrow$ GFG by Henzinger,Piterman, 2006

Lower bound LTL $\rightarrow$ GFG: straightforward adaption of Kupferman,Rosenberg 2011

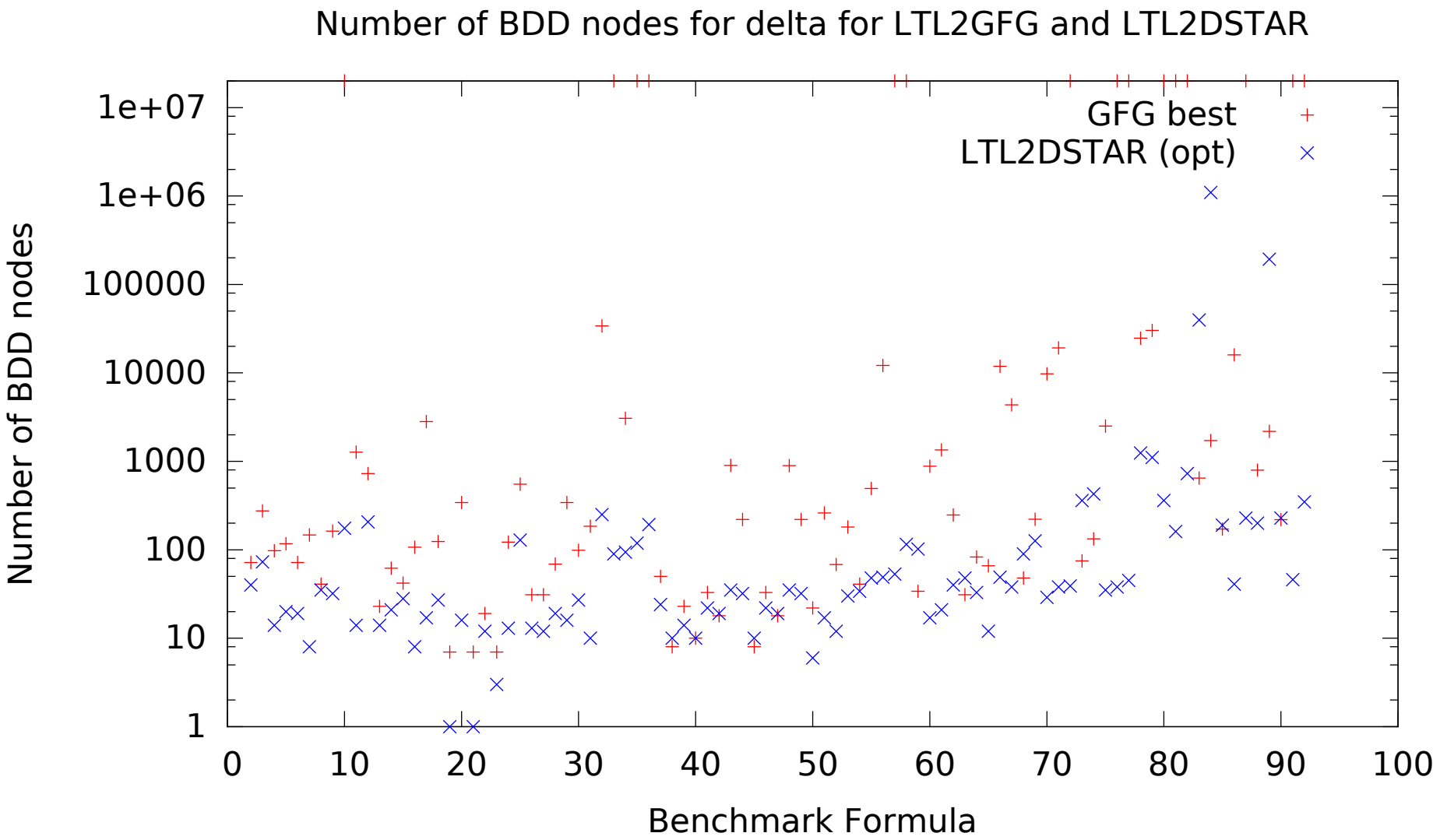
Implementations



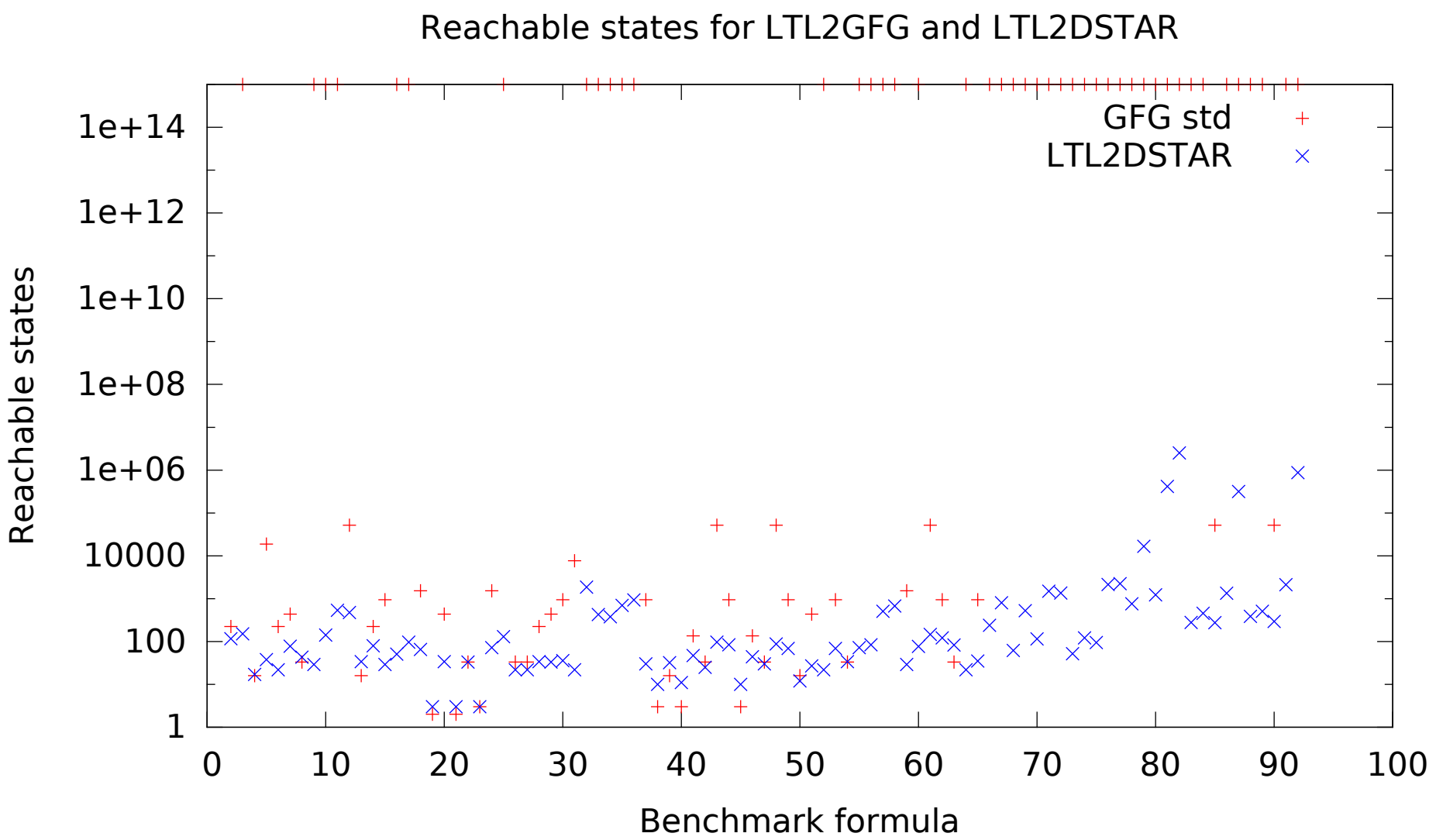
Size of symbolic BDD representation



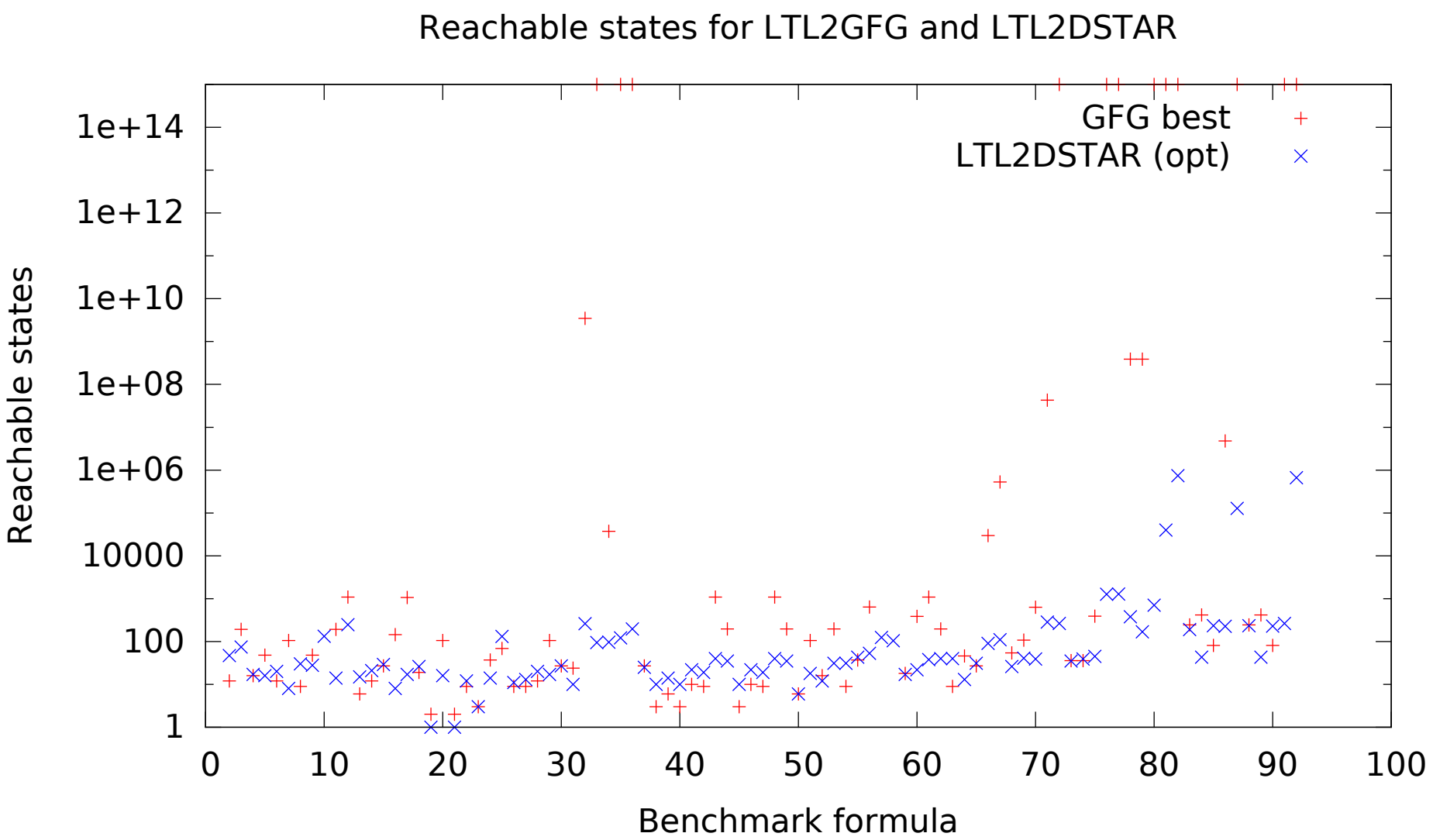
Size of symbolic BDD representation with heuristics



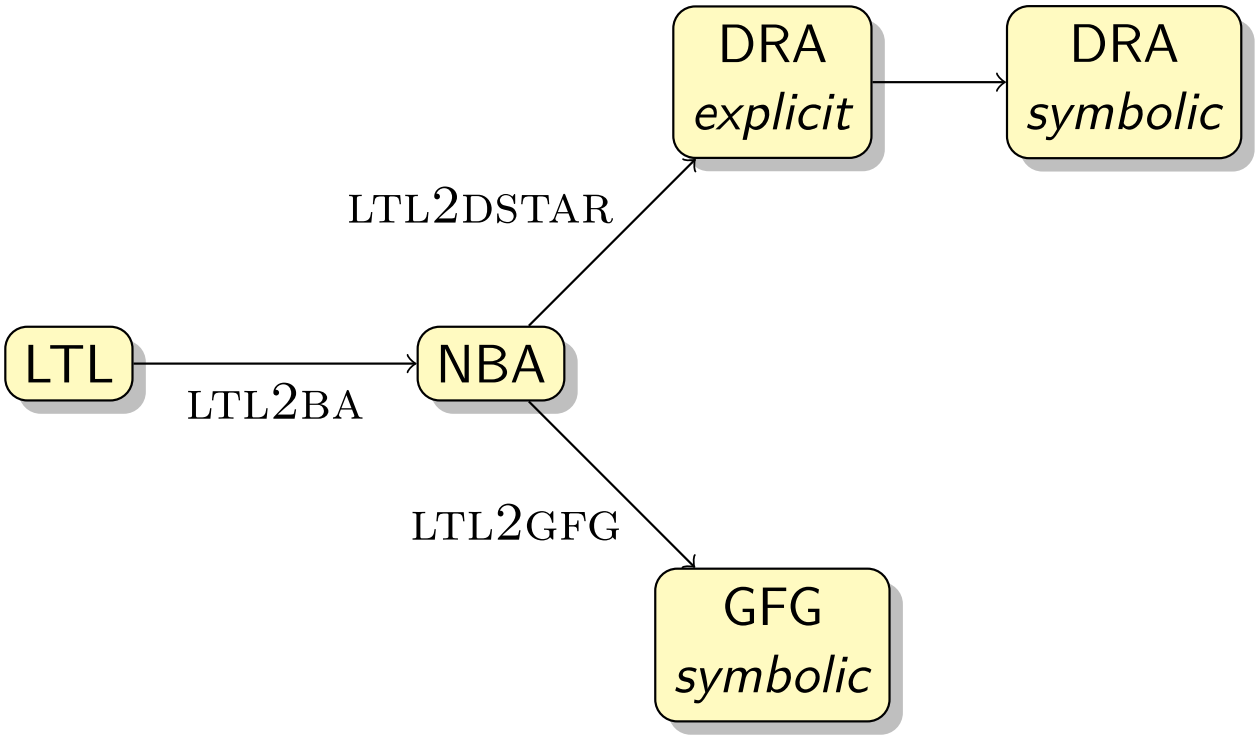
Reachable states



Reachable states with heuristics



Implementation in probabilistic model checker PRISM



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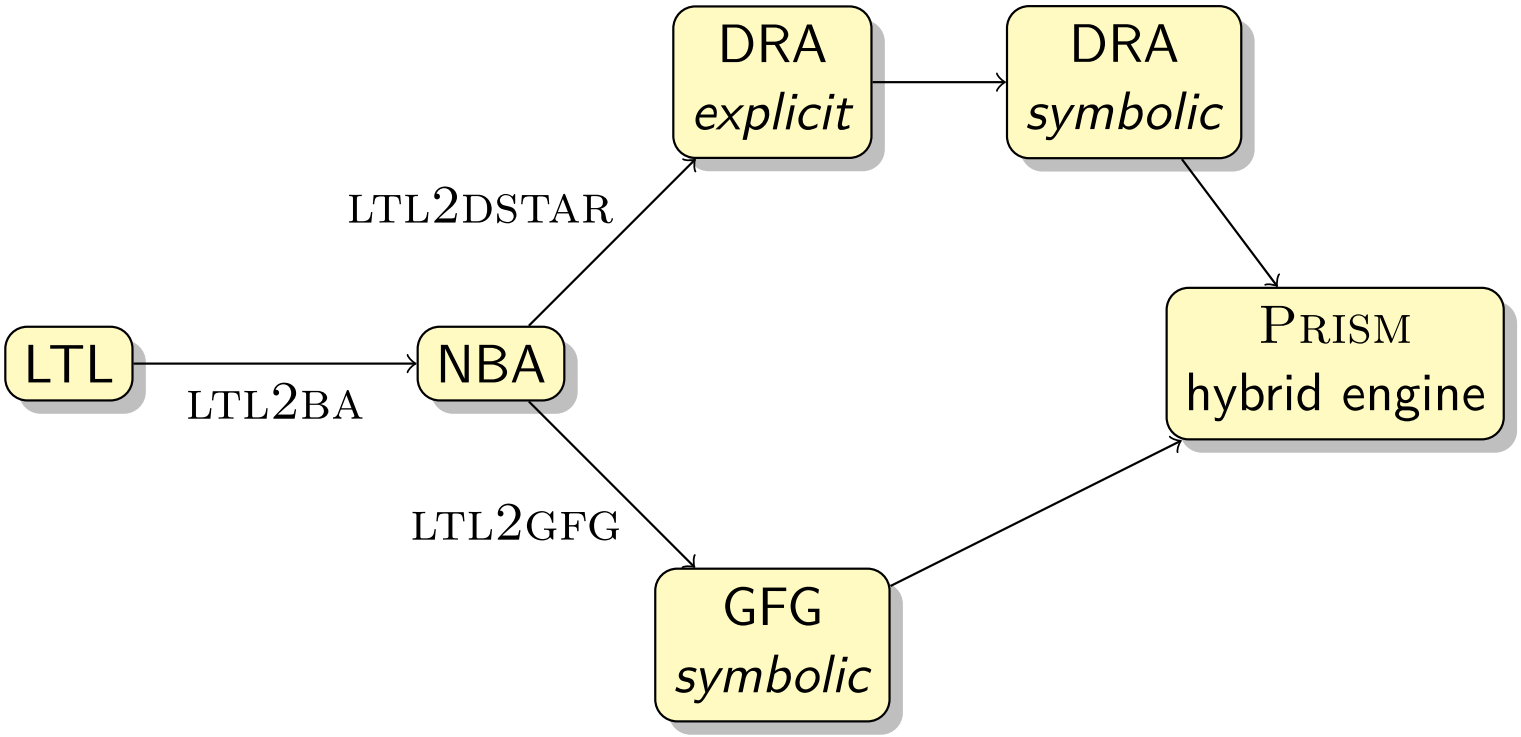

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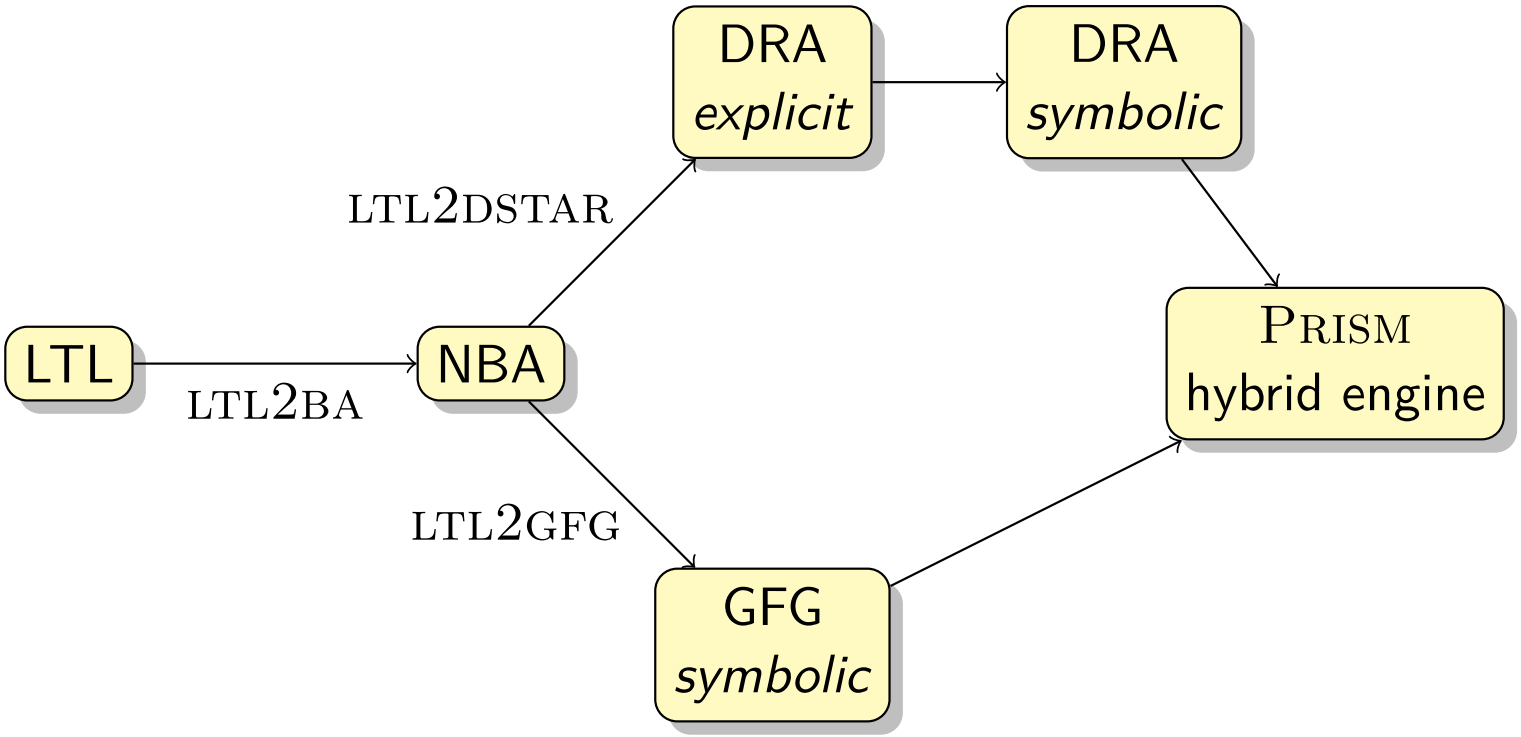

Case study: parts of WiFi carrier-sense protocol of IEEE 802.11 (Kwiatkowska, Norman and Sproston, 2002)

Benchmark: IEEE 802.11

$\Pr^{min}_{\mathcal{P}}(\varphi)$	time	
	LTL2GFG	PRISM std.
$\Diamond$Done	23.4 s	5.7 s
$\neg\Diamond\Box$error handling	38.4 s	0.7 s
$(\Box\Diamond\texttt{Backoff}_1) \rightarrow (\Box\Diamond\texttt{vuln}_1)$	63.4 s	2.9 s
$\Box(\texttt{colision} \rightarrow \Diamond\texttt{msg\_send}_1)$	43.6 s	3.1 s
$\Diamond\texttt{Done} \wedge \Box\#\texttt{collision} < 2$	97.6 s	42.9 s

Conclusion

- Established the use of *good-for-games automata* for model checking *Markov decision processes*

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- Evaluation of the transformation from NBA to GFG and the model checking process in PRISM

Thank you for your attention!

Questions or remarks?