Satisfiability of MTL and TPTL over Non-Monotonic Data Words[1]

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Data Words

P - finite set of atomic propositions

Definition. Data Words

$w = (P_0, d_0)(P_1, d_1)(P_2, d_2)\ldots$ (finite or infinite)

where:
- $P_i \subseteq \mathrm{P}$
- $d_i \in \mathbb{N}$

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Variants

Monotonic Data Words $d_i \leq d_{i+1}$

Timed Words $d_i \in \mathbb{R}$, $d_i \leq d_{i+1}$

Weather is non-monotonic

$$P = \{\text{sunny}, \text{cloudy}, \text{rain}, \text{fog}\}$$

$$w = (\text{sunny}, 7)(\text{cloudy}, 4)(\text{sunny}, 6)\ldots$$

MTL (Metric Temporal Logic)

TPTL (Timed Propositional Temporal Logic)

- extensions of LTL (Linear Temporal Logic)
- allow to specify properties of data values
- expressiveness and decidability widely studied on Timed Words and Monotonic Data Words

P - finite set of atomic propositions

Definition. MTL formulas

$$\varphi \;:=\; p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \varphi \mathsf{U}_{\mathrm{I}} \varphi$$

I interval of $\mathbb{Z}$ (open or closed), $p \in$ P

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Position of w: $i \in \mathbb{N}$

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- $(w, i) \models p$ iff $p \in P_i$.

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- $(w, i) \models \neg\varphi$ iff it is not the case that $(w, i) \models \neg\varphi$.

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- $(w, i) \models \varphi_1 \wedge \varphi_2$ iff $(w, i) \models \varphi_1$ and $(w, i) \models \varphi_2$.

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- $(w,i) \models p$ iff $p \in P_i$.
- $(w,i) \models \neg\varphi$ iff it is not the case that $(w,i) \models \neg\varphi$.
- $(w,i) \models \varphi_1 \wedge \varphi_2$ iff $(w,i) \models \varphi_1$ and $(w,i) \models \varphi_2$.
- $(w,i) \models \varphi_1 \mathsf{U}_{\mathrm{I}} \varphi_2$ iff there exist $j > i$ such that $(w,j) \models \varphi_2$,
  $(w,k) \models \varphi_1$ for all $i < k < j$,
  and $d_j - d_i \in \mathrm{I}$.

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Example

$$\varphi = p \,\mathsf{U}_{[2,4]} q$$

$$w = (\{p\},0)(\{p,q\},1)(\{p\},3)(\{q\},2)\ldots$$

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$(w,0) \models \varphi \quad \rightarrow \quad w \models \varphi$

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- $\varphi_1 \vee \varphi_2 := \neg(\neg\varphi_1 \wedge \neg\varphi_2)$

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- $\mathrm{X}_{\mathrm{I}}\varphi :=$ false$\mathrm{U}_{\mathrm{I}}\varphi$ - Next Modality

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- $\varphi_1 \vee \varphi_2 := \neg(\neg\varphi_1 \wedge \neg\varphi_2)$
- $\mathrm{true} := p \vee \neg p$
- $\mathrm{false} := \neg\mathrm{true}$
- $\mathrm{X}_{\mathrm{I}}\varphi := \mathrm{false}\mathrm{U}_{\mathrm{I}}\varphi$ - Next Modality
- $\mathrm{F}_{\mathrm{I}}\varphi := \mathrm{true}\mathrm{U}_{\mathrm{I}}\varphi$ - Finally Modality

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- $\mathrm{F}_{\mathrm{I}}\varphi := \mathrm{true}\mathrm{U}_{\mathrm{I}}\varphi$ - Finally Modality
- $\mathrm{G}_{\mathrm{I}}\varphi := \neg\mathrm{F}_{\mathrm{I}}\neg\varphi$ - Globally Modality

TPTL

P - finite set of atomic propositions

V - countable set of variables

Definition. TPTL formulas

$$\varphi \ := \ p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \varphi \mathsf{U} \varphi \mid x.\varphi \mid x \in \mathrm{I}$$

$p \in \mathrm{P}$, $\quad x \in \mathrm{V}$, $\quad \mathrm{I}$ interval of $\mathbb{Z}$ (open or closed)

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$$p \in \mathrm{P}, \quad x \in \mathrm{V}, \quad \mathrm{I} \text{ interval of } \mathbb{Z} \text{ (open or closed)}$$

Data Word: $w = (P_1, d_1)(P_2, d_2)(P_3, d_3)\ldots$
Position of w: $i \in \mathbb{N}$
Valuation function: $v : \mathrm{V} \to \mathbb{N}$

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- $(w, i, v) \models x.\varphi$ iff $(w, i, v[x \to d_i]) \models \varphi$.

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- $(w, i, v) \models x.\varphi$ iff $(w, i, v[x \to d_i]) \models \varphi$.
- $(w, i, v) \models x \in \mathrm{I}$ iff $d_i - v(x) \in \mathrm{I}$.

TPTL

Example

$$\text{MTL: } p\,\mathsf{U}_{[2,4]}\,q \quad \rightarrow \quad \text{TPTL: } x.\,p\,\mathsf{U}\,(q \wedge x \in [2,4])$$

TPTL

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Remark

There is an effective translation from MTL to TPTL[1]

1 → only 1 register variable

TPTL

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$$x.\mathsf{F}(a \wedge \mathsf{F}(b \wedge x \leq 2)) \qquad \text{(not expressible in MTL)}$$

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Example

$$x.\,\mathsf{F}(a \wedge \mathsf{F}(b \wedge x \leq 2)) \qquad \text{(not expressible in MTL)}$$

$$w = (c, \underbrace{1)(a,3)(c,4)(b,}_{1} 2) \dots$$

MTL and TPTL - overview

Results on Expressivity

- MTL and TPTL are equally expressive on Mon.Data Words [Alur, Henziger - 1993]
- TPTL is strictly more expressive than MTL on Timed words [Bouyer et al - 2010]

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Results on Satisfiability

- Satisfiability of MTL and TPTL over Mon.Data Words is decidable [Alur & Henziger - 1993]
- Satisfiability of MTL over finite Timed Words is decidable [Ouaknine & Worell - 2007]

FreezeLTL - reasoning on Data Words

- fragment of TPTL
- $x \in I$ is restricted to $x = 0$

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$$G[a \rightarrow x.(x{=}0 \, \mathsf{U} \, b)]$$

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- fragment of TPTL
- $x \in \mathrm{I}$ is restricted to $x = 0$

Example

$$G[a \rightarrow x.(x{=}0 \,\mathsf{U}\, b)]$$

$$w = (a,3)(c,3)(a,3)(b,2)\ldots$$

FreezeLTL - reasoning on Data Words

- fragment of TPTL
- $x \in I$ is restricted to $x = 0$

Example

$$G[a \to x.(x{=}0 \,\mathsf{U}\, b)]$$

$$w = (a,3)(c,3)(a,3)(b,2)\ldots$$

Remark

MTL and FreezeLTL are incomparable in expressivity:

- $\mathsf{F}_{\geq 0} b$ is not expressible in FreezeLTL
- $\mathsf{FFF}(x{=}0)$ is not expressible in MTL

FreezeLTL - overview

- Satisfiability of FreezeLTL[1] over FINITE Data Words is decidable
- Satisfiability of FreezeLTL[1] over INFINITE Data Words is undecidable [Demri, Lazic - 2009]

Question:

Are MTL and TPTL suitable logics for reasoning on Data Words?

MTL & TPTL over Data Words

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Answer:

Mostly NOT

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Finitary SAT over Data Words

INPUT: an MTL (TPTL, FreezeLTL) formula φ

QUESTION: is there a finite Data Word w such that $w \models \varphi$?

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Infinitary SAT over Data Words

INPUT: an MTL (TPTL, FreezeLTL) formula φ

QUESTION: is there an infinite Data Word w such that $w \models \varphi$?

DATA WORDS: overview

EXPRESSIVENESS: TPTL

MTL $\subsetneq$ TPTL, FreezeLTL $\subsetneq$ TPTL

MTL $\neq$ FreezeLTL

[Carapelle, Feng, Fernandez, Quaas - 2013]

Data Words

DATA WORDS: overview

EXPRESSIVENESS:

$$
\begin{array}{ccccc}
 & & \text{TPTL} & & \\
 & \subsetneq & & \supsetneq & \\
\text{MTL} & & \neq & & \text{FreezeLTL}
\end{array}
$$

[Carapelle, Feng, Fernandez, Quaas - 2013]

FreezeLTL[1]:
- Decidable FINITARY SAT
- Undecidable INFINITARY SAT

TPTL:
- Undecidable INFINITARY SAT
- FINITARY SAT?

MTL:
- INFINITARY SAT?
- FINITARY SAT?

DATA WORDS: overview

EXPRESSIVENESS: TPTL

MTL $\subsetneq$ TPTL $\supsetneq$ FreezeLTL

MTL $\neq$ FreezeLTL

[Carapelle, Feng, Fernandez, Quaas - 2013]

FreezeLTL[1]: • Decidable FINITARY SAT
• Undecidable INFINITARY SAT

TPTL: • Undecidable INFINITARY SAT
• FINITARY SAT? UNDECIDABLE

MTL: • INFINITARY SAT? UNDECIDABLE
• FINITARY SAT? UNDECIDABLE

Reduction of 2-counter machine **halting problem** and **recurring state problem**

Satisfiability for MTL & TPTL

IDEA: apply syntactical restrictions to regain decidability

Unary fragment: first introduced for FreezeLTL

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UnaryMTL: $\varphi \;:=\; p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathsf{X}_{\mathrm{I}}\varphi \mid \mathsf{F}_{\mathrm{I}}\varphi \mid \mathsf{G}_{\mathrm{I}}\varphi$

UnaryTPTL: $\varphi \;:=\; p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathsf{X}\varphi \mid \mathsf{F}\varphi \mid \mathsf{G}\varphi \mid x.\varphi \mid x \in \mathrm{I}$

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- SAT of UnaryTPTL (finitary and infinitary) UNDECIDABLE
- SAT of UnaryMTL (finitary and infinitary) UNDECIDABLE, but...

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- SAT of UnaryTPTL$-\{X\}$ is undecidable
- SAT of UnaryMTL$-\{X\}$?

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- SAT of UnaryTPTL$-\{X\}$ is undecidable $(\star)$
- SAT of UnaryMTL$-\{X\}$?

the proof for $(\star)$ uses formulas like $x.F(a \wedge F(b \wedge \ x \leq 2))$ - not expressible in MTL

Positive fragment

posMTL: $\varphi := p \mid \neg p \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid \varphi \mathsf{U}_{\mathrm{I}} \varphi$

posTPTL: $\varphi := p \mid \neg p \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid \varphi \mathsf{U} \varphi \mid x.\varphi \mid x \in \mathrm{I}$

Note: F and X can be obtained, G is not present

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Finite model property

For φ in posMTL or posTPTL ($\Rightarrow$ posFreezeLTL)

$$\varphi \text{ is satisfiable} \Leftrightarrow \varphi \text{ is satisfiable by a finite data word}$$

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- Infinitary SAT for posFreezeLTL[1] DECIDABLE

Satisfiability for MTL & TPTL

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φ is satisfiable $\Leftrightarrow$ φ is satisfiable by a finite data word

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- SAT of posTPTL (finitary and infinitary) UNDECIDABLE
- SAT of posMTL (finitary and infinitary) UNDECIDABLE

Satisfiability for MTL & TPTL

Positive Unary fragment

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posUnaryTPTL: $\varphi := \; p \mid \neg p \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid \mathsf{F}\varphi \mid \mathsf{X}\varphi \mid x.\varphi \mid x \in \mathrm{I}$

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Small model property

For φ in posUnaryMTL or posUnaryTPTL

φ is satisfiable $\Leftrightarrow$ φ is satisfiable by a data word w s.t. $|w| \leq |\varphi|$

Satisfiability for MTL & TPTL

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Small model property

For φ in posUnaryMTL or posUnaryTPTL

φ is satisfiable $\Leftrightarrow$ φ is satisfiable by a data word w s.t. $|w| \leq |\varphi|$

- SAT of posUnaryTPTL DECIDABLE, NP-complete
- SAT of posUnaryMTL DECIDABLE, NP-complete

Reduction Proof

$\mathcal{M}$: 2-counter machine with 4 states: $\{q_1, q_2, q_3, q_4\}$

Configuration: (q_i, C_1, C_2)

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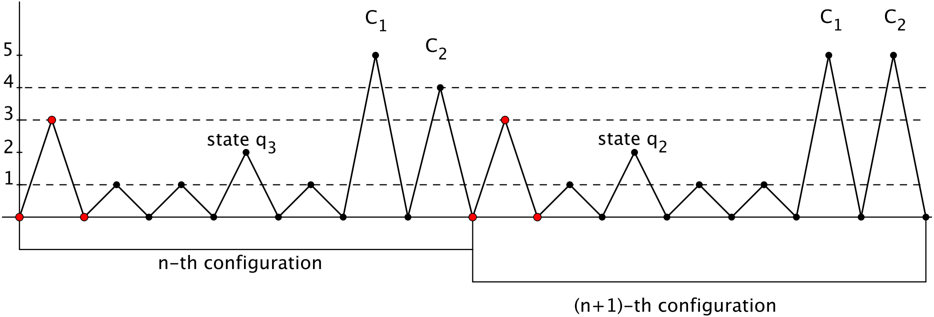

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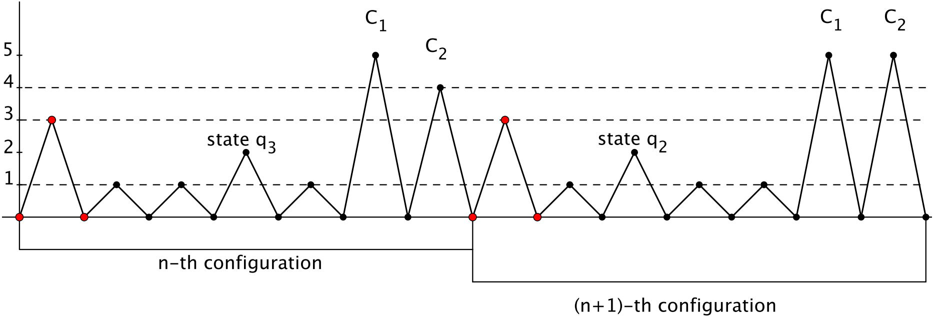

$$\mathsf{GF}(\mathsf{X}_{=3}\mathsf{true} \wedge \mathsf{XXX}_{=2}\mathsf{true})$$

Conclusion - Future work

Finitary SAT

	full	Unary	Unary - X	positive	posUnary
MTL	Σ_1^0-cpl.	Σ_1^0-cpl.	?	Σ_1^0-cpl.	NP-cpl.
TPTL	Σ_1^0-cpl.	Σ_1^0-cpl.	Σ_1^0-cpl.	Σ_1^0-cpl.	NP-cpl.
FreezeLTL[1]	NPR	NPR	NPR	NPR	NP-cpl.

Infinitary SAT

	full	Unary	Unary - X	positive	posUnary
MTL	Σ_1^1-cpl.	Σ_1^1-cpl.	?	Σ_1^0-cpl.	NP-cpl.
TPTL	Σ_1^1-cpl.	Σ_1^1-cpl.	Σ_1^1-cpl.	Σ_1^0-cpl.	NP-cpl.
FreezeLTL[1]	Π_1^0-cpl.	Π_1^0-cpl.	Π_1^0-cpl.	NPR	NP-cpl.

Conclusion - Future work

Finitary SAT

	full	Unary	Unary - X	positive	posUnary
MTL	Σ_1^0-cpl.	Σ_1^0-cpl.	?	Σ_1^0-cpl.	NP-cpl.
TPTL	Σ_1^0-cpl.	Σ_1^0-cpl.	Σ_1^0-cpl.	Σ_1^0-cpl.	NP-cpl.
FreezeLTL[1]	NPR	NPR	NPR	NPR	NP-cpl.

Infinitary SAT

	full	Unary	Unary - X	positive	posUnary
MTL	Σ_1^1-cpl.	Σ_1^1-cpl.	?	Σ_1^0-cpl.	NP-cpl.
TPTL	Σ_1^1-cpl.	Σ_1^1-cpl.	Σ_1^1-cpl.	Σ_1^0-cpl.	NP-cpl.
FreezeLTL[1]	Π_1^0-cpl.	Π_1^0-cpl.	Π_1^0-cpl.	NPR	NP-cpl.

Future work:

- UnaryMTL-X
- Restrict constraints expressivity instead of underlying LTL syntax