ω-rational Languages : High Complexity Classes vs. Borel Hierarchy

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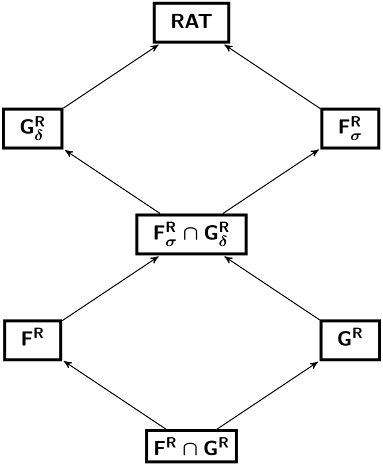
Motivation

- Initial problem : recognizing ζ-rational languages with non usual acceptance conditions
- For ω-languages, two main modes of acceptance : Büchi and Muller
- Can be expressed in the same formalism
- Study of "natural" acceptance conditions

ω-rational languages

- $\mathcal{L} \subseteq \Sigma^\omega$ is ω-rational iff $\mathcal{L} = \bigcup_{i=1}^{n} U_i V_i^\omega$ for U_i, V_i rational languages.
- $d(x, y) = 2^{-n}$ where $n = \min\{i \in \mathbb{N} : x_i \neq y_i\}$.
- F, G, F_σ and G_δ denote the sets of closed sets, open sets, countable unions of closed sets and countable intersections of open sets, respectively.
- RAT denotes the sets of ω-rational languages and F^R, G^R, F_σ^R and G_δ^R the previous classes intersected with RAT.

The Borel hierarchy



Finite automata

- Finite automaton $\mathcal{A} = (\Sigma, Q, T, I, \mathcal{F})$
  - Σ finite alphabet
  - Q finite set of states
  - $T \subseteq Q \times \Sigma \times Q$ set of transitions
  - $I \subseteq Q$ initial states
  - $\mathcal{F} \subseteq 2^Q$ table of accepting states
- A path p with label $x \in \Sigma^\omega$ is an infinite sequence of transitions $(p_i, x_i, p_{i+1}) \in T$
- A path p is initial if $p_0 \in I$

Usual acceptance conditions

- *Büchi acceptance condition* : a path is accepting iff some final state is visited infinitely often

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- What are the main points to define those acceptance conditions?
  - Some set of states depending of a path
  - A binary relation

Sets of states and relations

- Sets of states
  - $\mathtt{run}_{\mathcal{A}}(p) := \{q \in Q : \exists i > 0, p_i = q\}$
  - $\mathtt{inf}_{\mathcal{A}}(p) := \{q \in Q : \forall i > 0, \exists j \geq i, p_j = q\}$
  - $\mathtt{fin}_{\mathcal{A}}(p) := \mathtt{run}_{\mathcal{A}}(p) \setminus \mathtt{inf}_{\mathcal{A}}(p)$
  - $\mathtt{ninf}_{\mathcal{A}}(p) := Q \setminus \mathtt{inf}_{\mathcal{A}}(p)$
- Relations
  - Non-empty instersection $\sqcap$ $(S \sqcap T \Leftrightarrow S \cap T \neq \emptyset)$
  - Inclusion $\subseteq$
  - Equality $=$

Acceptance conditions

- *Acceptance condition* :

$$(c, \mathbf{R}) \in \{\mathtt{run}, \mathtt{inf}, \mathtt{fin}, \mathtt{ninf}\} \times \{\sqcap, \subseteq, =\}$$

- A word $x \in \Sigma^{\mathbb{N}}$ is recognized by $\mathcal{A}$ iff there exists an initial path p with label x such that

$$\exists F \in \mathsf{F}, c_{\mathcal{A}}(p) \ \mathbf{R} \ F$$

- $\mathcal{L}_{\mathcal{A}}^{(c,\mathbf{R})} := \left\{x \in \Sigma^{\mathbb{N}} : x \text{ is recognized by } \mathcal{A} \text{ under condition } (c, \mathbf{R})\right\}$

- Büchi is $(\mathtt{inf}, \sqcap)$ and Muller is $(\mathtt{inf}, =)$

- All conditions induce ω-rational languages (they can be expressed in MSO logic)

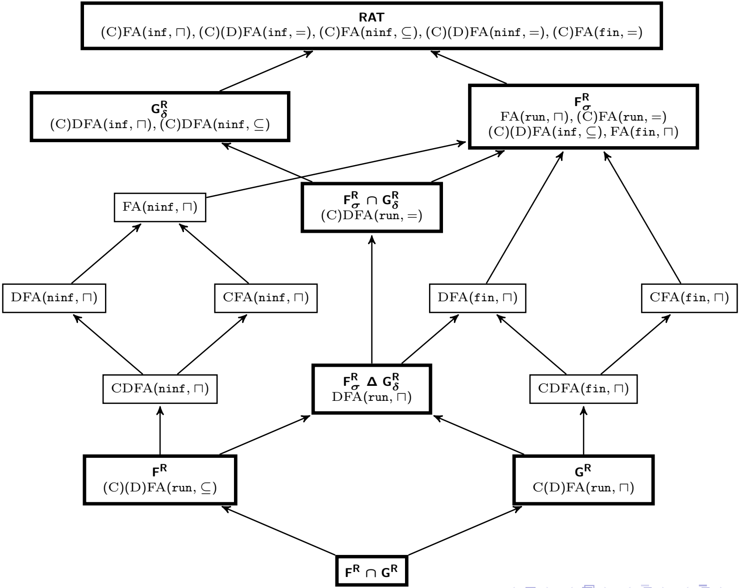
Classes of languages : determinism and completeness

- There exists languages recognized under Büchi acceptance conditions only by non-deterministic automaton.
- There exists languages recognized under $(\mathtt{run}, \sqcap)$ acceptance conditions only by non-complete automaton.
- Let $\mathit{cond} \in \{\mathtt{run}, \mathtt{inf}, \mathtt{fin}, \mathtt{ninf}\} \times \{\sqcap, \subseteq, =\}$,
  - $\mathrm{FA}(\mathit{cond}) = \{\mathcal{L}_{\mathcal{A}}^{\mathit{cond}},\ \mathcal{A} \text{ is a } \mathrm{FA}\}$
  - $\mathrm{DFA}(\mathit{cond}) = \{\mathcal{L}_{\mathcal{A}}^{\mathit{cond}},\ \mathcal{A} \text{ is a } \mathrm{DFA}\}$
  - $\mathrm{CFA}(\mathit{cond}) = \{\mathcal{L}_{\mathcal{A}}^{\mathit{cond}},\ \mathcal{A} \text{ is a } \mathrm{CFA}\}$
  - $\mathrm{CDFA}(\mathit{cond}) = \{\mathcal{L}_{\mathcal{A}}^{\mathit{cond}},\ \mathcal{A} \text{ is a } \mathrm{CDFA}\}$

State of the art

	$\sqcap$	$\subseteq$	$=$
`run`	Landweber '69	Hartmanis Stearns '67	Staiger Wagner '74
`inf`	Büchi '60	Landweber '69	Muller '63
`fin`	Litovsky Staiger '97	Dennunzio *et al.* '12 (partial results)	Dennunzio *et al.* '12 (partial results)
`ninf`	Moriya and Yamasaki '88 Dennunzio *et al.* '12	Moriya and Yamasaki '88 Dennunzio *et al.* '12	Dennunzio *et al.* '12

State of the art



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- $\mathtt{fin}'_{\mathcal{A}}(p) := \mathtt{run}'_{\mathcal{A}}(p) \smallsetminus \mathtt{inf}_{\mathcal{A}}(p)$
- What about $\mathtt{fin}$ and $\mathtt{fin}'$?

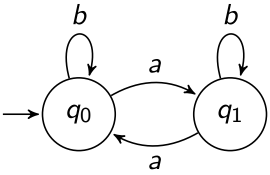
Witness languages

- $\mathcal{L}_{k,n}^{\Sigma,\alpha}$ denotes the language $\{x \in \Sigma^\omega : |x|_\alpha = k \pmod{n}\}$ where Σ is a finite alphabet, $\alpha \in \Sigma$, $k \geq 0$, $n > 0$
- $\tilde{\mathcal{L}}_{k,n}^{\Sigma,\alpha}$ denotes the language $\mathcal{L}_{k,n}^{\Sigma,\alpha} + (\Sigma^*\alpha)^\omega$
- In our examples, $\Sigma = \{a, b\}$

$\texttt{fin}$ vs $\texttt{fin}'$

Proposition

$\mathcal{L}_{1,2}^{\Sigma,a} \in \mathrm{CDFA}(\texttt{fin}', \sqcap)$



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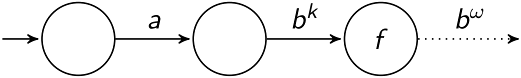

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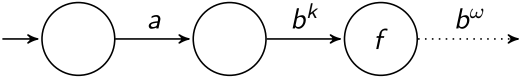

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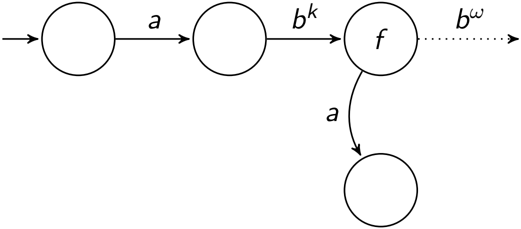

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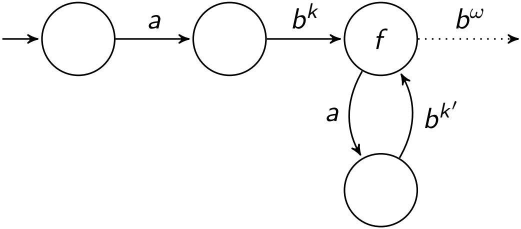
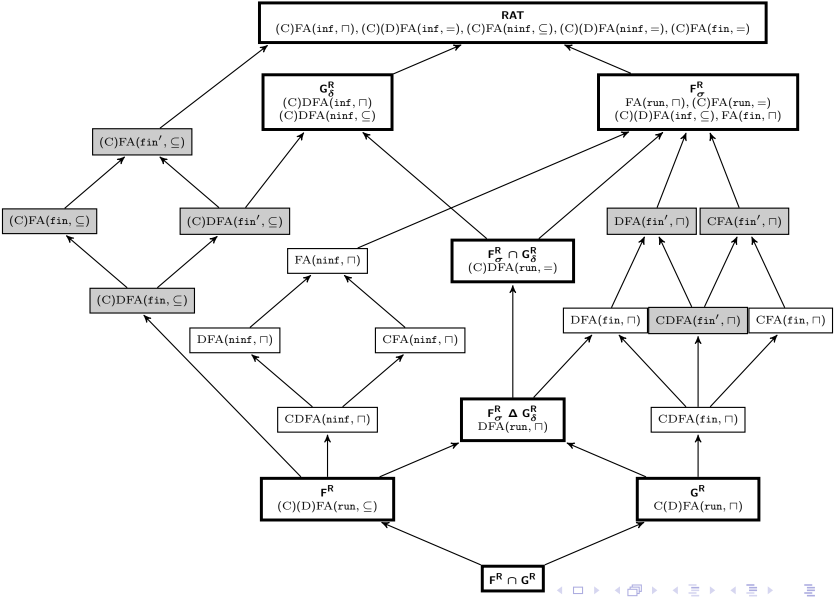


The hierarchy



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Open question (DLT '12)

Does $CDFA(\mathtt{fin}, =)$ *equal* RAT*?*

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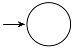

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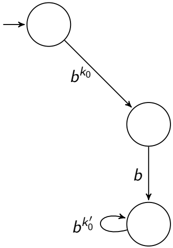

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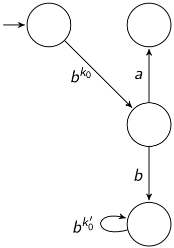

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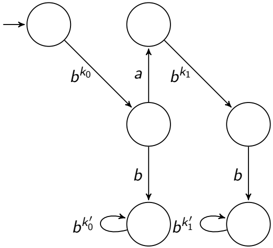

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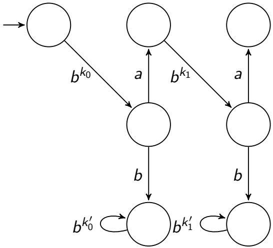

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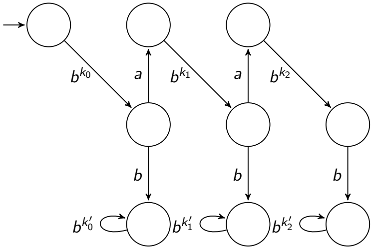

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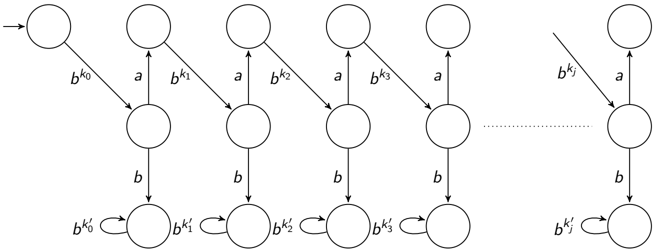

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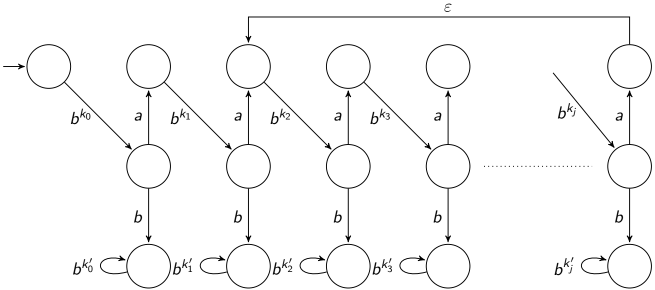

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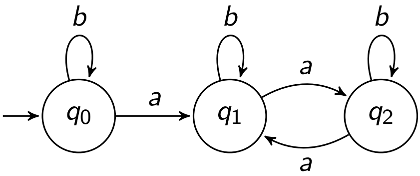

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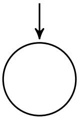

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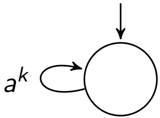

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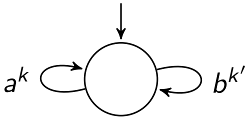

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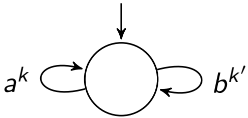

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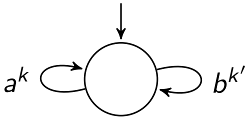
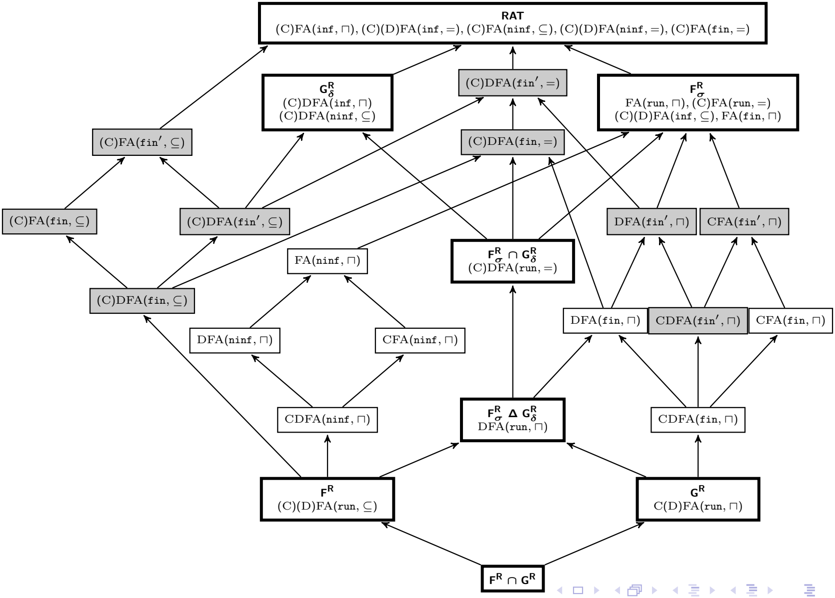


$$x = a^\omega \in \mathcal{L} \Rightarrow \texttt{fin}'_{\mathcal{A}}(p_x) = \emptyset \in \mathcal{F}$$
$$y = (a^k b^{k'})^\omega \notin \mathcal{L} \text{ but } \texttt{fin}'_{\mathcal{A}}(p_y) = \emptyset \in \mathcal{F}$$

Hierarchy with $(\mathtt{fin}, =)$ and $(\mathtt{fin}', =)$



Conclusion and future work

- Complete hierarchy
- New high complexity classes

- Closure properties according to classical operation on languages
- Characterization of the new classes
- Decidability of the membership
- Descriptional complexity