Characterisation of the State Spaces of Live and Bounded Marked Graph Petri Nets

Eike Best and Raymond Devillers

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Analysis and synthesis of Petri nets

- **Analysis** (Esparza et al.)
  Deduce behavioural properties of a Petri net
- **Synthesis** (Rozenberg et al.)
  Derive a Petri net realising a labelled transition system

A line of research (Darondeau et al.)
Classes of Petri nets vs. classes of labelled transition systems

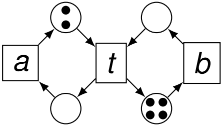
This paper relates

- marked graph Petri nets
- to a subclass of labelled transition systems

Marked graphs have been applied in manufacturing,
in controller synthesis, and in asynchronous hardware design

A live and bounded marked graph

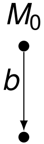
M_0

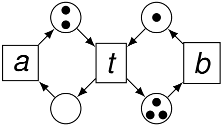


A marked graph Petri net

and its initial marking M_0

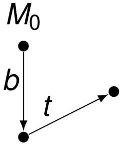
A live and bounded marked graph

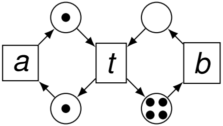




after executing *b*

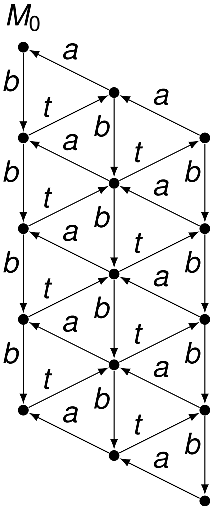
A live and bounded marked graph

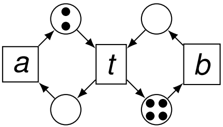




after executing *bt*

A live and bounded marked graph



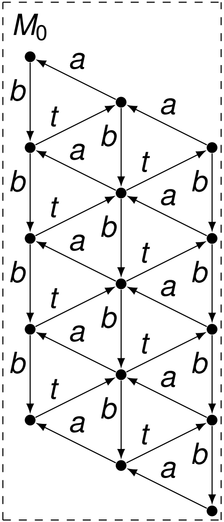


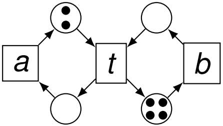
A marked graph Petri net

and its reachability graph..

..which has several nice properties:

It is **finite**

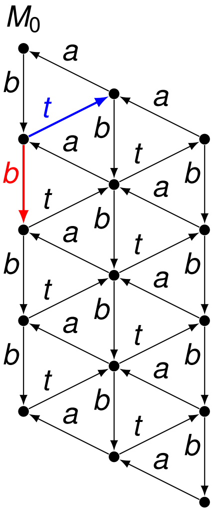


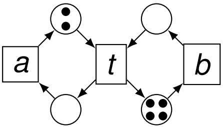


Finiteness

..due to the boundedness of the net

It is **deterministic**

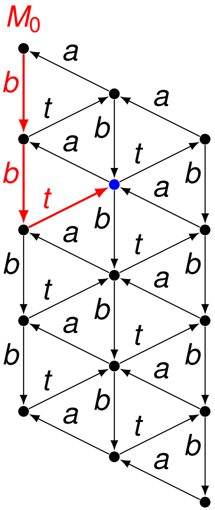


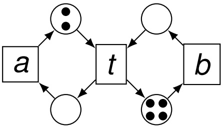


Determinism If a state enables b and t, leading to different states, then $b \neq t$

.. true because the reachability graph comes from a Petri net

It is **totally reachable**

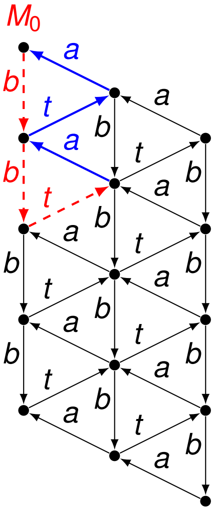


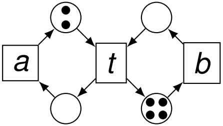


Total reachability Every state is reachable from the initial state M_0

.. true by the definition of reachability graph

It is **reversible**

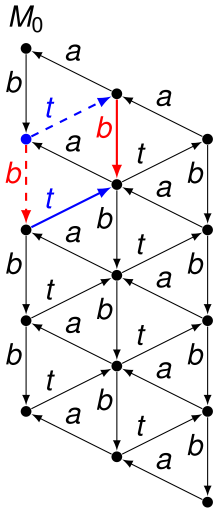


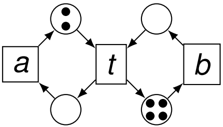


Reversibility The initial state is reachable from every reachable state

.. true (for marked graphs) by liveness and boundedness

It is **persistent**



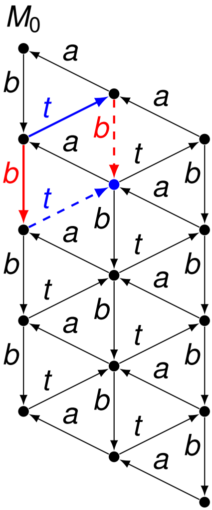


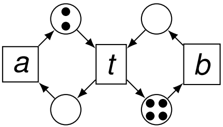
Persistency If a state enables b and t for $b \neq t$, then it also enables bt and tb

.. true by the marked graph property

also called *strong confluence*

It is **backward persistent**

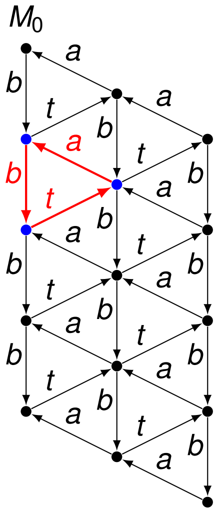


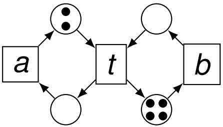


Backward persistency
If a state backward enables b and t for $b \neq t$, from two reachable states, then it also backward enables bt and tb

.. true by the marked graph property

It satisfies the **P1 property**





The Parikh 1 property
In a small cycle, every firable transition occurs exactly once

.. true by the marked graph property

Note: $M_0[bbttaa\rangle M_0$ is not small

Properties of live and bounded marked graphs

Definition A labelled transition system is **nice** if

- it is finite
- deterministic
- totally reachable
- reversible
- persistent
- backward persistent
- and satisfies the P1 property of small cycles

Theorem

Commoner, Genrich, Holt, Even, Lautenbach, Pnueli (1968..)

The reachability graph of a live and bounded
marked graph Petri net is nice

Main result of this paper – A converse

Theorem (LATA' 2014)

If a labelled transition system is nice, then it is the reachability graph of some live and bounded marked graph Petri net

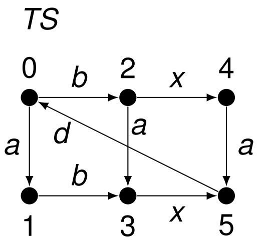
Moreover:
There is a unique minimal marked graph realising it

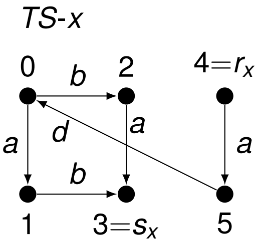
Moreover:
Place bounds can be calculated from the lts

Proof: Constructively

Reducing a labelled transition system *TS*

Let x be a label and let TS-x be obtained from TS by erasing all x-labelled arrows





Lemma: If TS is nice, then TS-x

- is connected and acyclic
- has a unique minimal element r_x
- has a unique maximal element s_x

Computing a net from a labelled transition system *TS*

The states of *TS*-x can be partitioned into

- $NE(x)$ = states not enabling x (including r_x)
- $EN(x)$ = states enabling x (including s_x)

Lemma: Every maximal state in $NE(x)$ equals s_a, for some $a \neq x$

- Let the labels of *TS* be the transitions of the net
- For a label x, pick a maximal state $s = s_a \in NE(x)$
- Create a place p with incoming transition a and outgoing transition x
- Let the initial marking of p be the number of a's on a path from r_x to s_0
  Lemma: It doesn't matter which path
- Exhaustively perform this construction to obtain a net

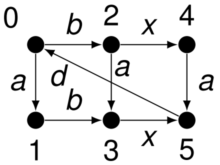
Theorem: This net is a live and bounded marked graph realising *TS*

Moreover: it is side-condition-free, unique, and minimal

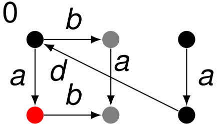
Moreover: the bound of p is the number of a's on a path from s_a to s_x

A worked example with initial state $s_0 = 0$

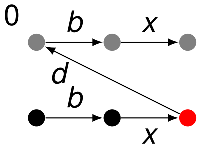
TS



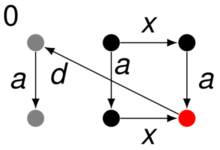
TS-x



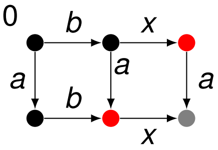
TS-a



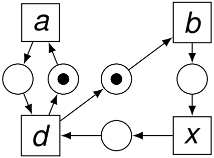
TS-b



TS-d



Solution:



States in $EN(x)$ are drawn in gray
Maximal states in $NE(x)$ are drawn in red
The places correspond to the red states

Necessity of niceness

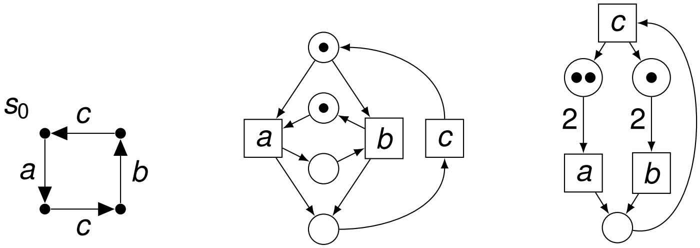
If all but one niceness properties are satisfied for some lts *TS*

then no live and bounded marked graph
has a reachability graph isomorphic to *TS*

Also: The uniqueness of minimal solutions may fail

Niceness minus the P1 property

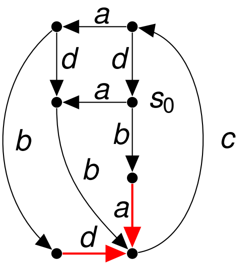
The lts shown below satisfies all niceness properties except P1

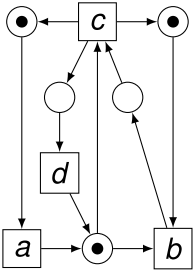


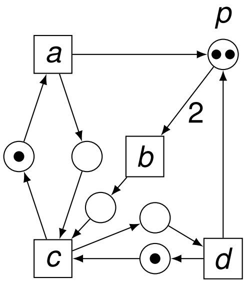
There are two different minimal non-marked graph solutions

Niceness minus backward persistency

The lts shown below satisfies all niceness properties except backward persistency







There are two different minimal non-marked graph solutions

Concluding remarks

Done:
A characterisation of ‘nice’ labelled transition systems in terms of a structurally defined class of Petri nets

Applications:
E.g., a fast and direct synthesis algorithm

Other possible uses:
Help in proving open conjectures

Extensions:
Modify / relax lts properties or net classes

E.g., what if backward persistency is omitted / relaxed?
Answer: doesn’t work easily / we don’t know, respectively