Reachability Analysis with State-Coherent Automata

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- Introduction
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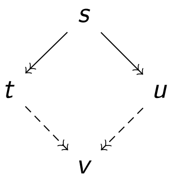
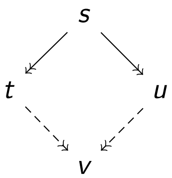
Motivation

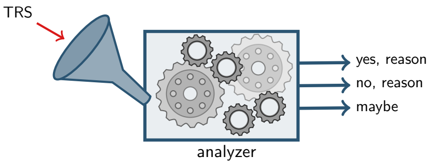
- Confluence of Term Rewrite Systems
  e.g. $\{\mathsf{F}(x,x) \to \mathsf{A}, \mathsf{G}(x) \to \mathsf{F}(x, \mathsf{G}(x)), \mathsf{C} \to \mathsf{G}(\mathsf{C})\}$



Motivation

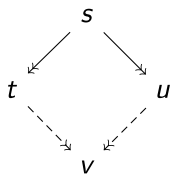
- Confluence of Term Rewrite Systems
  e.g. $\{\mathsf{F}(x,x) \to \mathsf{A}, \mathsf{G}(x) \to \mathsf{F}(x,\mathsf{G}(x)), \mathsf{C} \to \mathsf{G}(\mathsf{C})\}$
- Automation (CSI)
  but can we trust the tool?

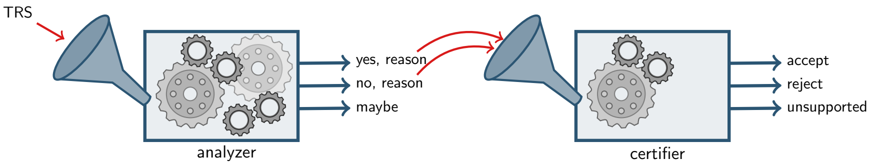




Motivation

- Confluence of Term Rewrite Systems
  e.g. $\{\mathsf{F}(x,x) \to \mathsf{A}, \mathsf{G}(x) \to \mathsf{F}(x,\mathsf{G}(x)), \mathsf{C} \to \mathsf{G}(\mathsf{C})\}$
- Automation (CSI)
  but can we trust the tool?
- Certification (CeTA)
  two tools are better than one
  especially if one is proved correct





Application: Non-Confluence

$\mathcal{R}: \qquad \mathsf{F}(x,x) \to \mathsf{A} \qquad \mathsf{G}(x) \to \mathsf{F}(x,\mathsf{G}(x)) \qquad \mathsf{C} \to \mathsf{G}(\mathsf{C})$

Rewriting

- rules $\ell \to r \in \mathcal{R}$, where $\ell, r \in \mathcal{T}(\mathcal{F},\mathcal{V})$ and $\mathcal{V}\mathrm{ar}(r) \subseteq \mathcal{V}\mathrm{ar}(\ell)$

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Idea

- $\mathcal{R}^*(\mathsf{A})$ and $\mathcal{R}^*(\mathsf{G}(\mathsf{A}))$ are disjoint:
- $\mathcal{R}^*(\mathsf{A}) = \{\mathsf{A}\}$
- $\mathcal{R}^*(\mathsf{G}(\mathsf{A})) = \{\mathsf{G}(\mathsf{A}), \mathsf{F}(\mathsf{A},\mathsf{G}(\mathsf{A})), \mathsf{F}(\mathsf{A},\mathsf{F}(\mathsf{A},\mathsf{G}(\mathsf{A}))), \dots\}$

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$\Rightarrow$ Overapproximate $\mathcal{R}^*(t)$

Setup

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If $\mathcal{L}(\mathcal{A})$ is closed under $\to_{\mathcal{R}}$ and $t \in \mathcal{L}(\mathcal{A})$ then $\mathcal{R}^*(t) \subseteq \mathcal{L}(\mathcal{A})$.

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Question

Given a BTA $\mathcal{A}$, and a TRS $\mathcal{R}$, is $\mathcal{L}(\mathcal{A})$ closed under $\to_{\mathcal{R}}$?

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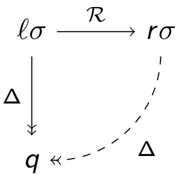
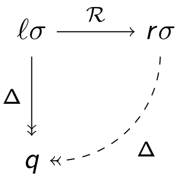
Compatibility



Definition

$\mathcal{A}$ is compatible with $\mathcal{R}$ if for all $\sigma : \mathcal{V} \to Q$ and $\ell \to r \in \mathcal{R}$, $\ell\sigma \to_\Delta^* q \in Q$ implies $r\sigma \to_\Delta^* q$.

Compatibility



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Theorem (Genet 1998)

If $\mathcal{A}$ is compatible with $\mathcal{R}$ then $\mathcal{L}(\mathcal{A})$ is closed under $\to_{\mathcal{R}}$ if $\mathcal{R}$ is left-linear or $\mathcal{A}$ is deterministic.

A Limitation

$$\mathcal{R}: \qquad \mathsf{F}(x,x) \to \mathsf{B} \qquad \mathsf{B} \to \mathsf{A}$$

Claim

$\{\mathsf{F}(\mathsf{A},\mathsf{A}), \mathsf{B}, \mathsf{A}\}$ is not accepted by a deterministic, compatible automaton.

A Limitation

$\mathcal{R}: \quad F(x,x) \to B \quad B \to A$

Claim

$\{F(A,A), B, A\}$ is not accepted by a deterministic, compatible automaton.

- by compatibility, $F(A,A), B, A$ are accepted in the same state $q \in Q_f$

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- by compatibility, $\mathsf{F}(\mathsf{A}, \mathsf{A}), \mathsf{B}, \mathsf{A}$ are accepted in the same state $q \in Q_f$
- by determinism, $\{\mathsf{F}(q, q) \to q, \mathsf{B} \to q, \mathsf{A} \to q\} \subseteq \Delta$

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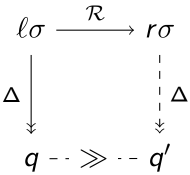
- $\mathcal{T}(\{\mathsf{F}, \mathsf{B}, \mathsf{A}\}) \subseteq \mathcal{L}(\mathcal{A})$ ∎

State-Compatibility

Definition

Let $\gg \subseteq Q \times Q$.

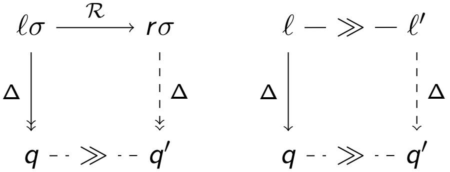
State-Compatibility



Definition

Let $\gg \subseteq Q \times Q$.
$(\mathcal{A}, \gg)$ is state-compatible with $\mathcal{R}$ if for all $\sigma : \mathcal{V} \to Q$ and $\ell \to r \in \mathcal{R}$,
$\ell\sigma \to_{\Delta}^{*} q$ implies $r\sigma \to_{\Delta}^{*} q'$ for some q' with $q \gg q'$.

State-Compatibility



Definition

Let $\gg \subseteq Q \times Q$.
$(\mathcal{A}, \gg)$ is state-compatible with $\mathcal{R}$ if for all $\sigma : \mathcal{V} \to Q$ and $\ell \to r \in \mathcal{R}$, $\ell\sigma \to_\Delta^* q$ implies $r\sigma \to_\Delta^* q'$ for some q' with $q \gg q'$.
$(\mathcal{A}, \gg)$ is state-coherent if $q \gg q'$ and $q \in Q_f$ implies $q' \in Q_f$, and for all $f(q_1, \ldots, q_n) \to q \in \Delta$ and $q_i \gg q_i'$, $f(q_1, \ldots, q_i', \ldots, q_n) \to q' \in \Delta$ for some some q' with $q \gg q'$.

Soundness

Theorem

If $(\mathcal{A}, \gg)$ is state-coherent and state-compatible with $\mathcal{R}$ then $\mathcal{L}(\mathcal{A})$ is closed under $\to_{\mathcal{R}}$ if $\mathcal{R}$ is left-linear or $\mathcal{A}$ is deterministic.

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$$\begin{array}{ccc} \mathcal{L}(\mathcal{A}) \ni & C[\ell\tau] \\ & \mathcal{R} \Big\downarrow \\ & C[r\tau] \end{array}$$

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$$\begin{array}{ccccccccc} \mathcal{L}(\mathcal{A}) \ni & C[\ell\tau] & \xrightarrow{\Delta}\!\!\!\!\rightarrow & C[\ell\sigma] & \xrightarrow{\Delta}\!\!\!\!\rightarrow & C[q] & \xrightarrow{\Delta}\!\!\!\!\rightarrow & q_\bullet & \in Q_f \\ & \mathcal{R}\downarrow & & & & & & & \\ & C[r\tau] & & & & & & & \end{array}$$

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- $Q = Q_f = \{0,1\}$, $1 \gg 0$
- $\Delta = \{\mathsf{A} \to 0, \mathsf{B} \to 1, \mathsf{F}(0,0) \to 1\}$ ■

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Theorem (Completeness)

Let $\mathcal{A}$ be deterministic and trim, and $\mathcal{L}(\mathcal{A})$ be closed under $\to_{\mathcal{R}}$. Then $(\mathcal{A},\gg)$ is state-coherent and state-compatible with $\mathcal{R}$ for some $\gg$.

Decidability

Theorem

Let $\mathcal{A}$ be a BTA and $\mathcal{R}$ be a TRS. It is decidable whether $\mathcal{L}(\mathcal{A})$ is closed under $\rightarrow_{\mathcal{R}}$.

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2. compute minimal relation $\gg$ that makes $\mathcal{A}$ state-compatible with $\mathcal{R}$ and state-coherent
   - return no if $\ell\sigma \to_{\Delta}^{*} q$ but not $r\sigma \to_{\Delta}^{*} q'$ for any $q' \in Q$; otherwise, add $q \gg q'$
   - return no if $f(q_1, \dots, q_n) \to q \in \Delta$, $q_i \gg q_i'$ but not $f(q_1, \dots, q_i', \dots, q_n) \to q' \in \Delta$ for any $q' \in Q$; otherwise, add $q \gg q'$
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   - return no if $q \in Q_f$, $q \gg q'$ but not $q' \in Q_f$
3. return yes

Implementation

IsaFoR + CeTA

- soundness and completeness: formalized in IsaFoR, executable
- also formalized tree automata intersection and emptiness check

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⇒ CeTA can now certify tree-automata based non-confluence proofs

CSI

- CSI can produce state-coherent, state-compatible automata for non-confluence proofs

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Summary

Done

- state-compatible, state-coherent automata
- simple, sound and complete
- most important results formalized in IsaFoR
- non-confluence support in CSI, CeTA
- ...

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- ...

Future work

- match-bounds
- do completion directly on state-compatible automata

Summary

Done

- state-compatible, state-coherent automata
- simple, sound and complete
- most important results formalized in IsaFoR
- non-confluence support in CSI, CeTA
- ...

Future work

- match-bounds
- do completion directly on state-compatible automata

Thank you!