Extended Two-Way Ordered Restarting Automata for Picture Languages

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1. Introduction

The restarting automaton models the linguistic technique of analysis by reduction (Jančar et. al., 1995).

Various classes of formal languages have been characterized by certain types of restarting automata:

REG	:	R(1)-automata (Mráz, 2001)
REG	:	det-RR(1) (Reimann, 2007)
DCFL	:	det-mon-R(R)(W)(W) (Jančar et. al., 1999)
CFL	:	mon-R(R)WW-automata (Jančar et. al., 1999)
CRL	:	det-R(R)WW-automata (Niemann et. al., 1998)
GCSL	:	wmon-R(R)WW-automata (Jurdziński et. al., 2004)

Many studies have extended grammars and automata from word languages to picture languages, e.g.:

- 4-way finite automata (Blum, Hewitt 1967),
- isometric array grammars (Rosenfeld 1971),
- matrix grammars (Siromoney et. al. 1972),
- tiling systems and automata (Giammarresi, Restivo 1992),
- Sudoku-deterministically recognizable languages (Borchert, Reinhardt 2007).

The class REC of recognizable picture languages has been identified as a central class:

- various nice characterizations and
- good closure properties,
- but it contains NP-complete languages, that is, NP-complete membership problems (Lindgren et. al. 1998).

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Quest for two-dimensional automata with the following properties:

- intuitive model, that is, easy way of designing algorithms (automata) for interesting languages,
- membership problems decidable in polynomial time,
- restricted to word languages, only the regular languages should be accepted,
- nice closure properties.

Several models have been proposed recently:

- the restarting tiling automaton (Průša, Mráz, CIAA 2012)
- the Sgraffito automaton (Průša, Mráz, DLT 2012)
- the deterministic 2-dimensional 3-way ordered restarting automaton (Mráz, Otto, SOFSEM 2014)

Here: the determ. 2-dim. extended 2-way ordered restarting automaton (det-2D-x2W-ORWW-automaton).

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2. Deterministic 3-Way ORWW-Automata

A picture P over Σ is a finite two-dimensional array of symbols from Σ.

$\mathrm{row}(P)$ ($\mathrm{col}(P)$) denotes the number of rows (columns) of P,

$P(i,j)$ is the symbol at position (i,j), $1 \leq i \leq \mathrm{row}(P)$, $1 \leq j \leq \mathrm{col}(P)$.

By $\Sigma^{m,n}$ we denote the set of all pictures of size $m \times n$ over Σ,

and $\Sigma^{*,*}$ is the set of all pictures over Σ.

Let $\mathcal{S} = \{\vdash, \dashv, \top, \bot, \#\}$ be a set of five special markers (sentinels).

In order to enable an automaton to detect the border of P easily, we define the boundary picture $\widehat{P}$ over $\Sigma \cup \mathcal{S}$ of size $(m+2) \times (n+2)$:

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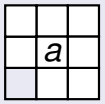
$\vdash$						$\dashv$
$\vdots$			P			$\vdots$
$\vdash$						$\dashv$
$\#$	$\bot$	$\bot$	$\cdots$	$\bot$	$\bot$	$\#$

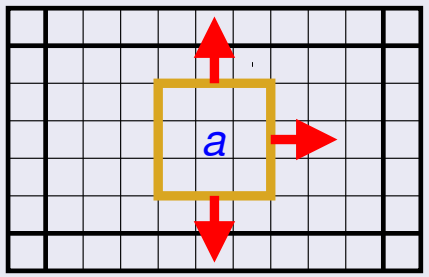
Definition 1

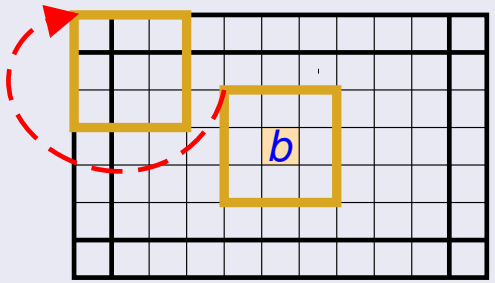
Deterministic two-dimensional three-way ordered RWW-automaton $M = (Q, \Sigma, \Gamma, \mathcal{S}, q_0, \delta, >)$:

- Q *is a finite set of states containing the initial state* q_0,
- Σ *is a finite input alphabet,* Γ *is a finite tape alphabet containing* Σ *such that* $\Gamma \cap \mathcal{S} = \emptyset$
- $>$ *is a partial ordering on* Γ
- δ *is the transition function,*

  for each $C =$ 

  it holds at most one of the following:
  - $\delta(q, C) = (q', \mathrm{R})$ – *move right step*
  - $\delta(q, C) = (q', \mathrm{D})$ – *move down step*
  - $\delta(q, C) = (q', \mathrm{U})$ – *move up step*
  - $\delta(q, C) = b$ *and* $a > b$ – *rewrite and restart*





Theorem 2

$\mathcal{L}(\text{det-2D-3W-ORWW}) \subseteq \text{DTIME}((\text{size}(P))^2)$.

Theorem 3 (M., O., SOFSEM 2014)

$\mathcal{L}(\text{det-2D-3W-ORWW}) \cap \Sigma^{1,*} = \text{REG}(\Sigma)$, *that is, the* det-2D-3W-ORWW*-automaton only accepts regular word languages.*

Theorem 4 (M., O., SOFSEM 2014)

The det-2D-3W-ORWW*-automaton can simulate the deterministic Sgraffito automaton, which in turn is known to be able to simulate*

- *(alternating) four-way finite automata,*
- *deterministic four-way one-marker automata,*
- *and to accept all sudoku-deterministically recognizable languages.*

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Remarks

- The det-2D-3W-ORWW-automata clearly favour vertical over horizontal movements. Accordingly, $\mathcal{L}$(det-2D-3W-ORWW) is not closed under transposition.

- It could happen that a det-2D-3W-ORWW-automaton M does not terminate on some input picture, as it may get stuck on a column, just moving up and down.

To avoid this, it is required explicitly that M halts on all input pictures!

This can be realized by providing a simple pattern, e.g.,
$\text{up}^* - \text{down}^* - \text{up}^* - \text{down}^*$, such that on each column,
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3. Deterministic Extended 2-Way ORWW-Automata

Definition 5

A deterministic two-dimensional extended two-way ordered RWW-automaton (det-2D-x2W-ORWW*-automaton*) *is given through a 7-tuple*

$$M = (Q, \Sigma, \Gamma, \mathcal{S}, q_0, \delta, >),$$

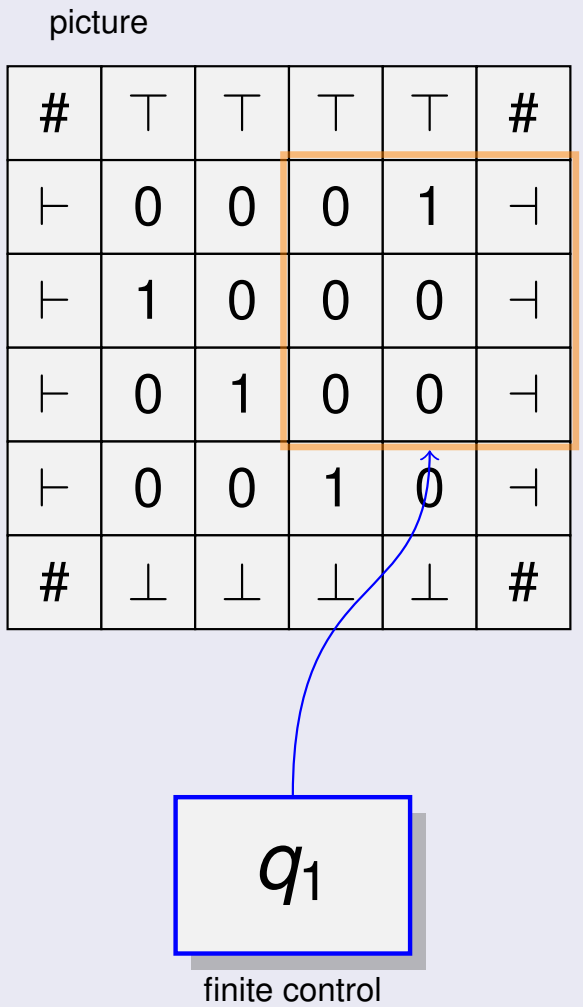
where all components are defined as for det-2D-3W-ORWW*-automata. However, the set of possible head movements is restricted to*

$$\mathcal{H} = \{\mathrm{R}, \mathrm{D}\},$$

where the move-right and move-down steps are extended as follows:

Definition 5 (cont.)

Extended Move-Right Step:



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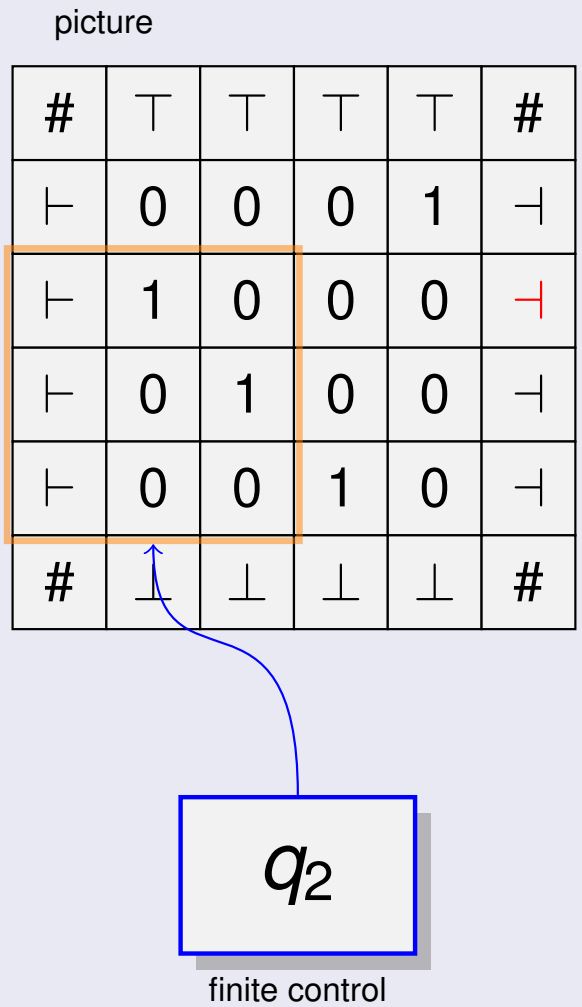

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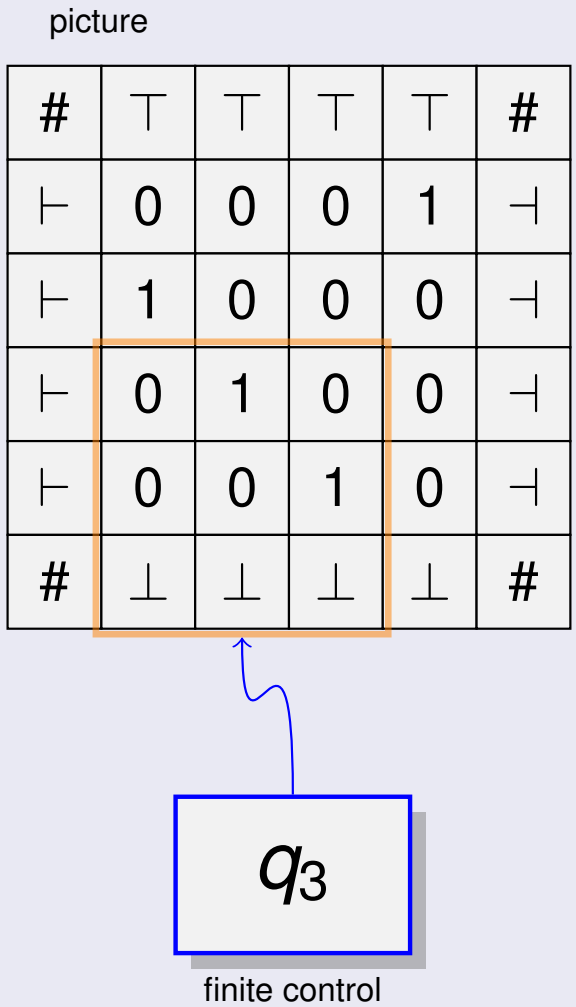
Extended Move-Down Step:



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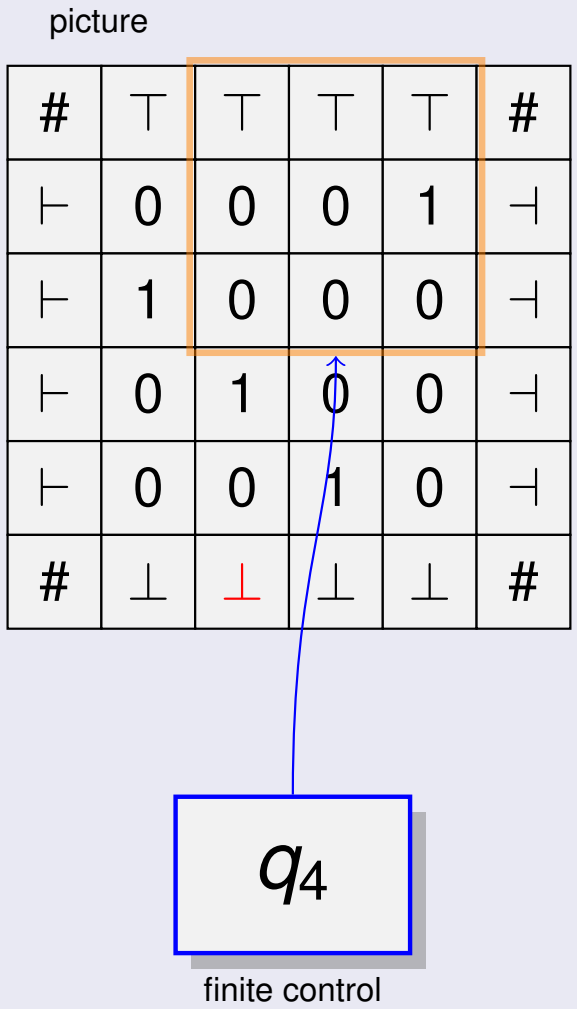

Definition 5 (cont.)

In any cycle, M can only use extended move-right or extended move-down steps, but not both!

M is a stateless det-2D-x2W-ORWW-automaton (stl-det-2D-x2W-ORWW-automaton) if it has just a single state. For such an automaton, the components Q and q_0 are suppressed.

Corollary 6

When restricted to one-dimensional input, then the det-2D-x2W-ORWW*-automaton just accepts the regular word languages. This also holds for the stateless variant.*

Theorem 7

$\mathcal{L}(\text{det-2D-x2W-ORWW}) \subseteq \text{DTIME}((\text{size}(P))^3)$.

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Example 1

Let $\Sigma = \{0, 1\}$, and let

$$L_{\text{perm}} = \{\, P \in \Sigma^{*,*} \mid \text{row}(P) = \text{col}(P) \geq 1, \text{ each row and each column contains exactly one symbol 1} \,\}.$$

We describe a stl-det-2D-x2W-ORWW-automaton M_{perm} for L_{perm}.

Let

$$\Gamma = \Sigma \cup \{0', 1', 0'_1, 0'', 1'', 0''_1\},$$

and let

$$1 > 0 > 1' > 0' > 0'_1 > 1'' > 0'' > 0''_1.$$

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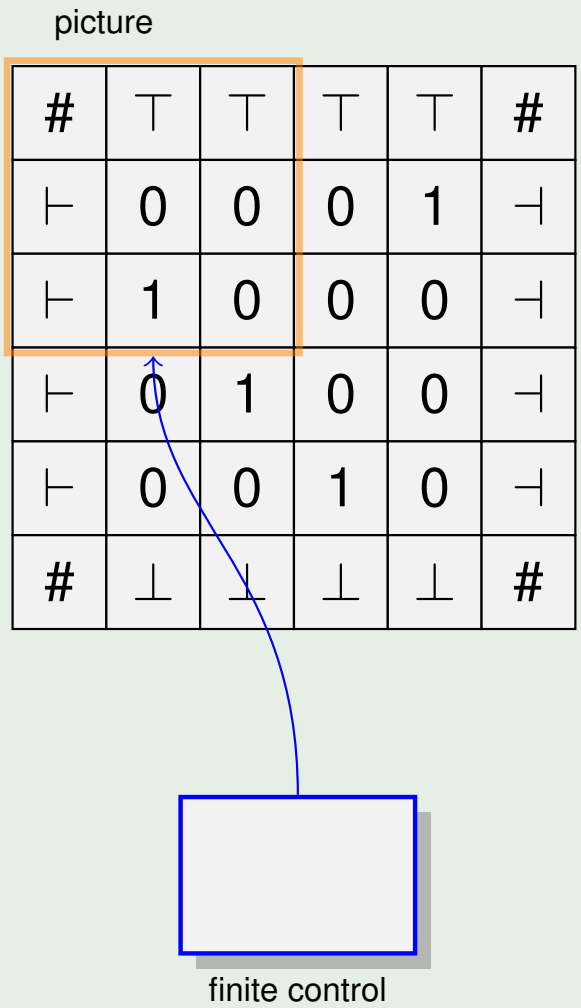

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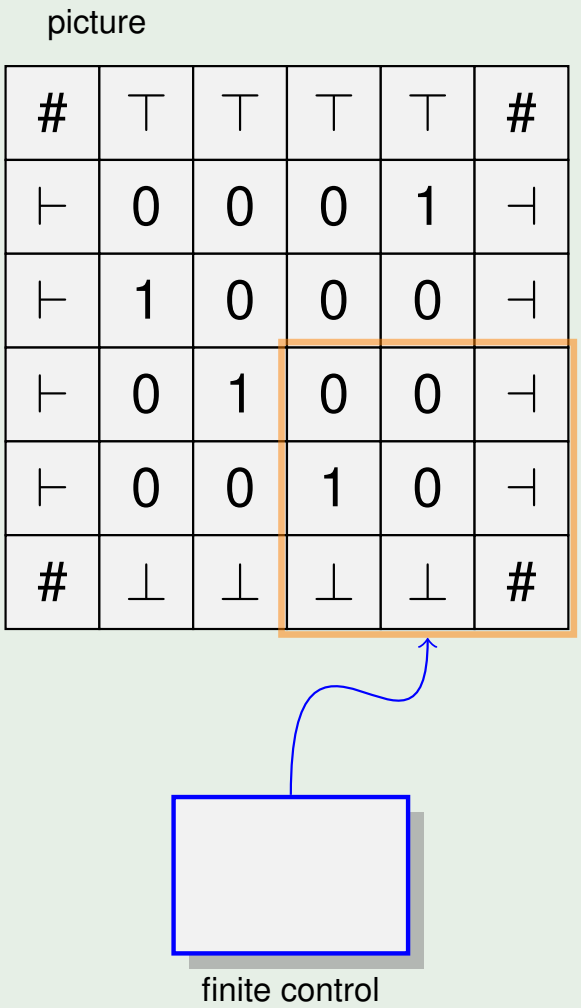

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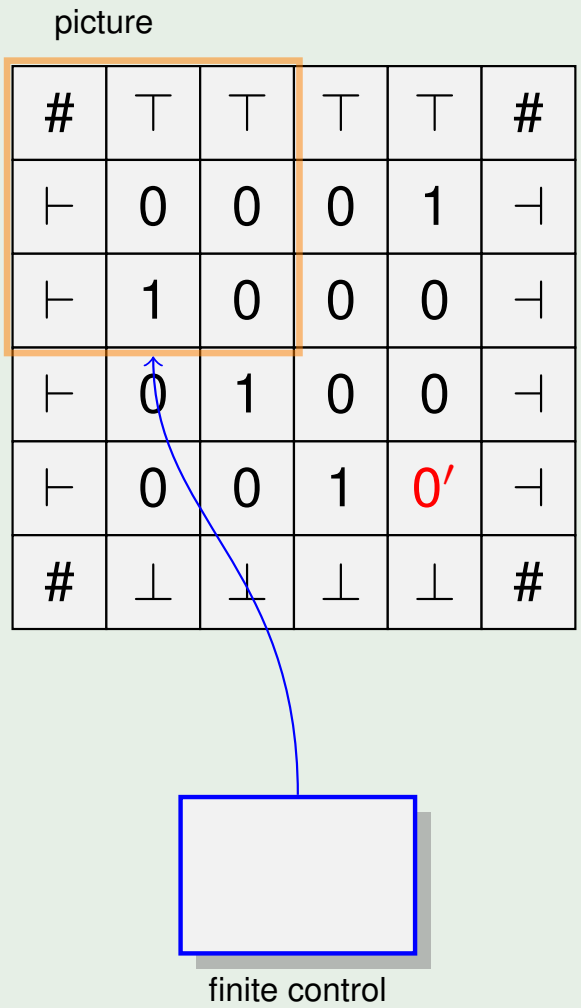

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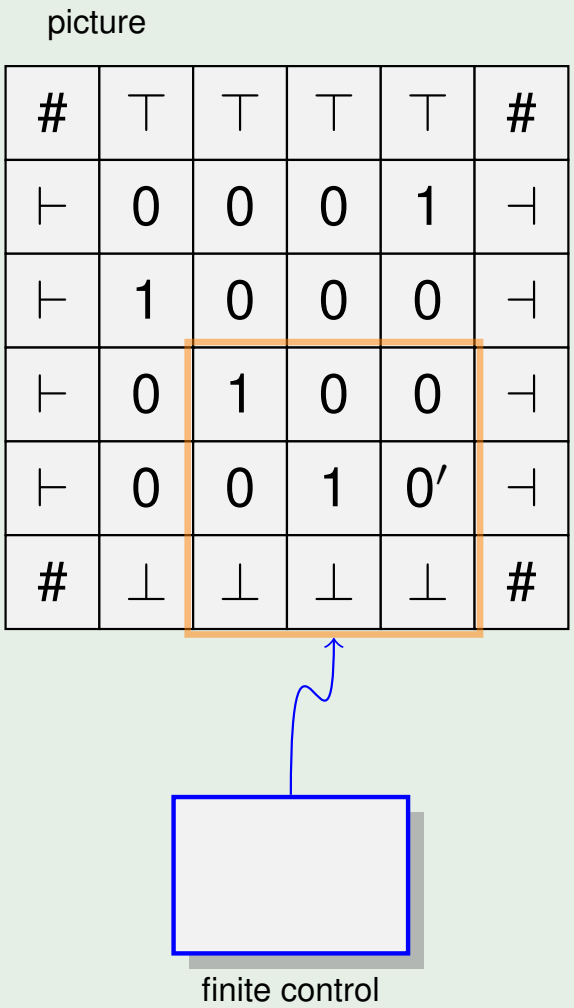

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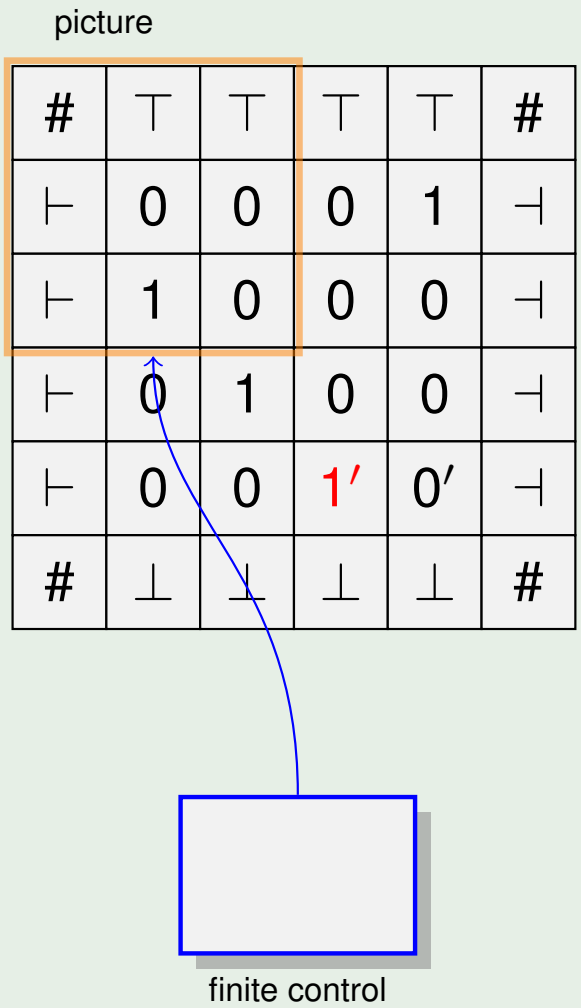

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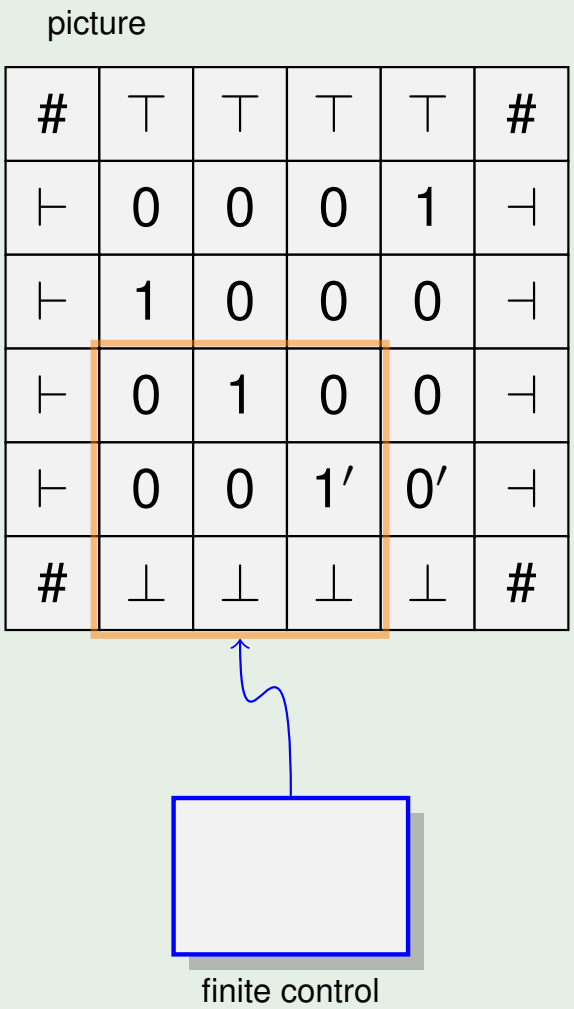

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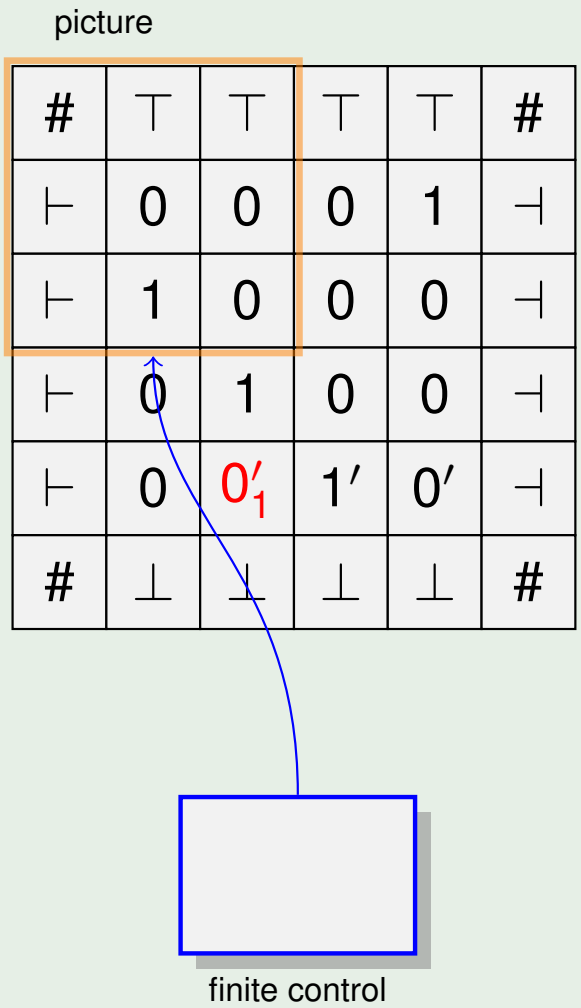

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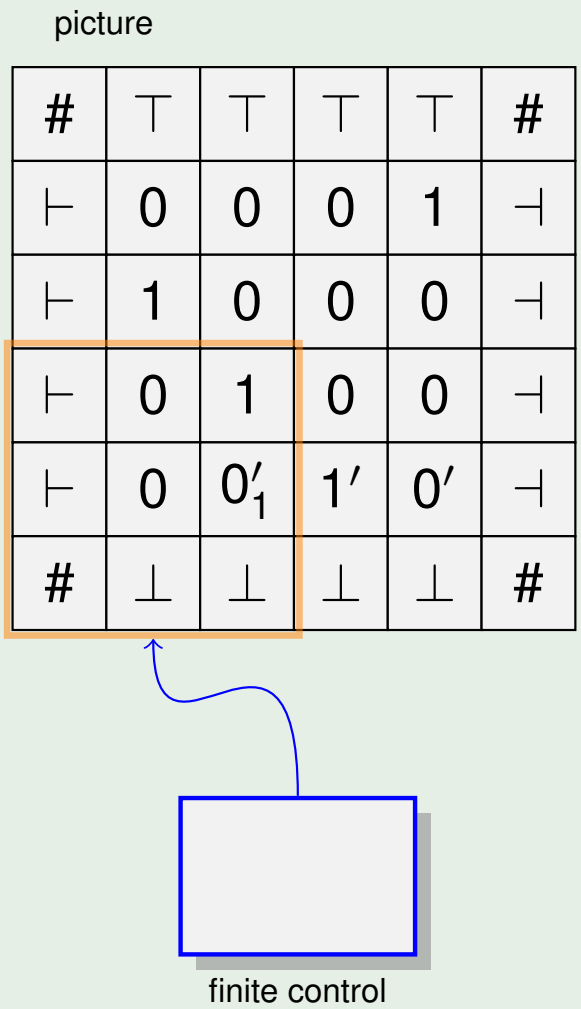

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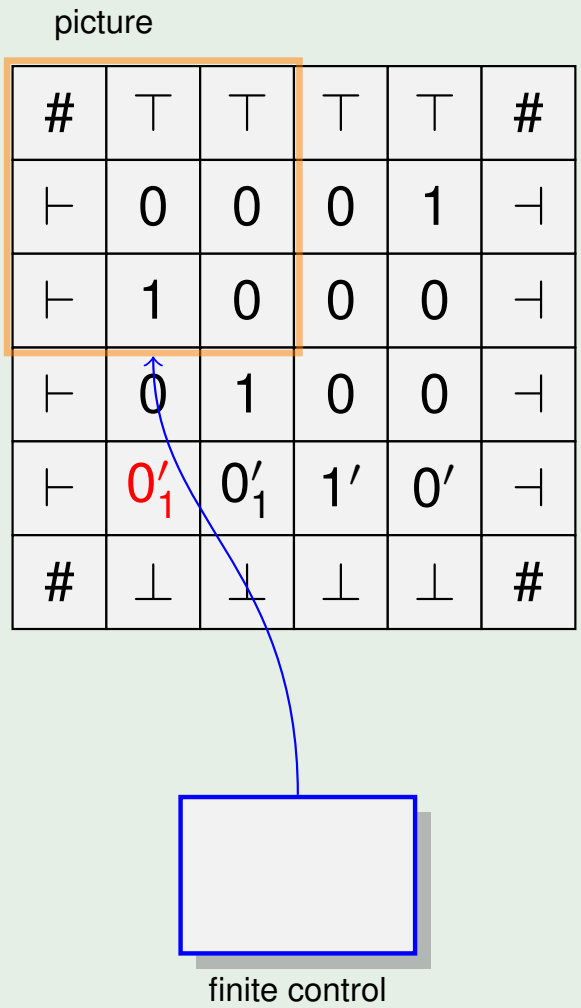

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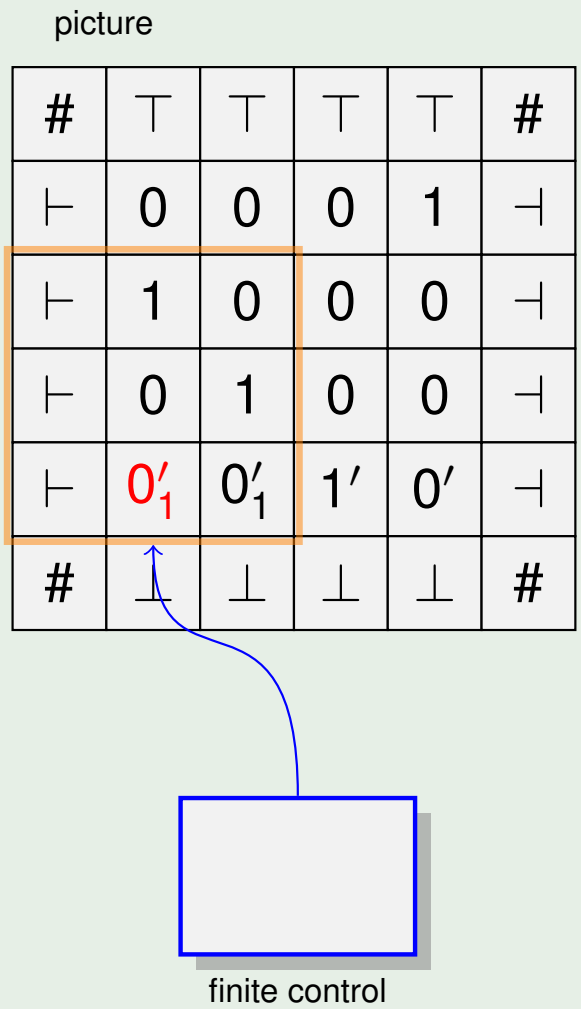

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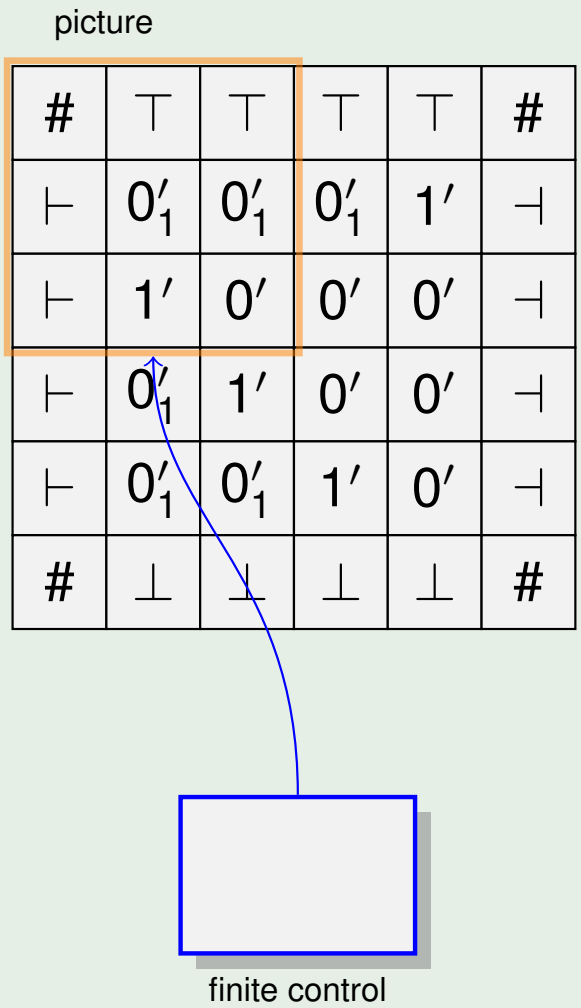

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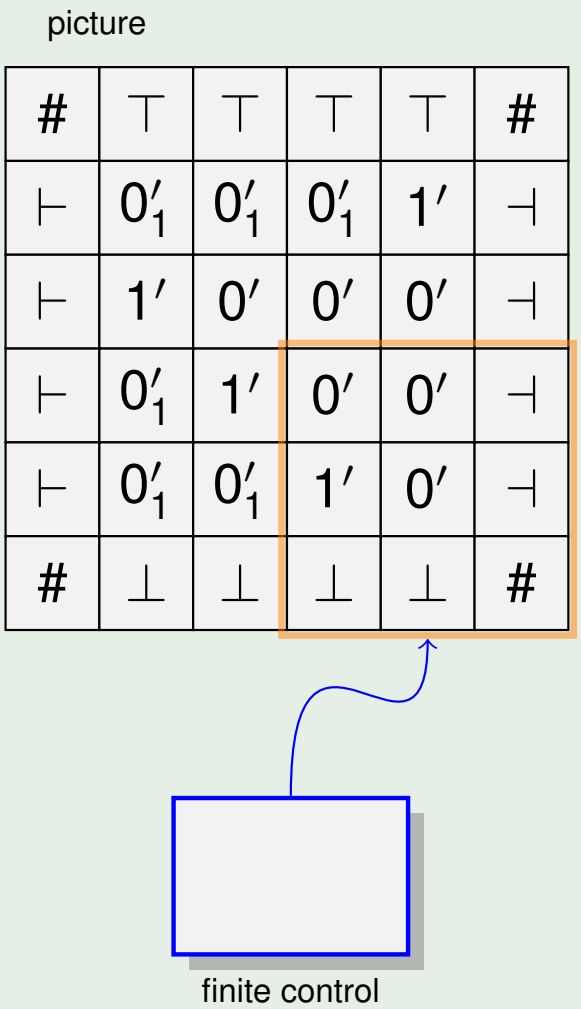

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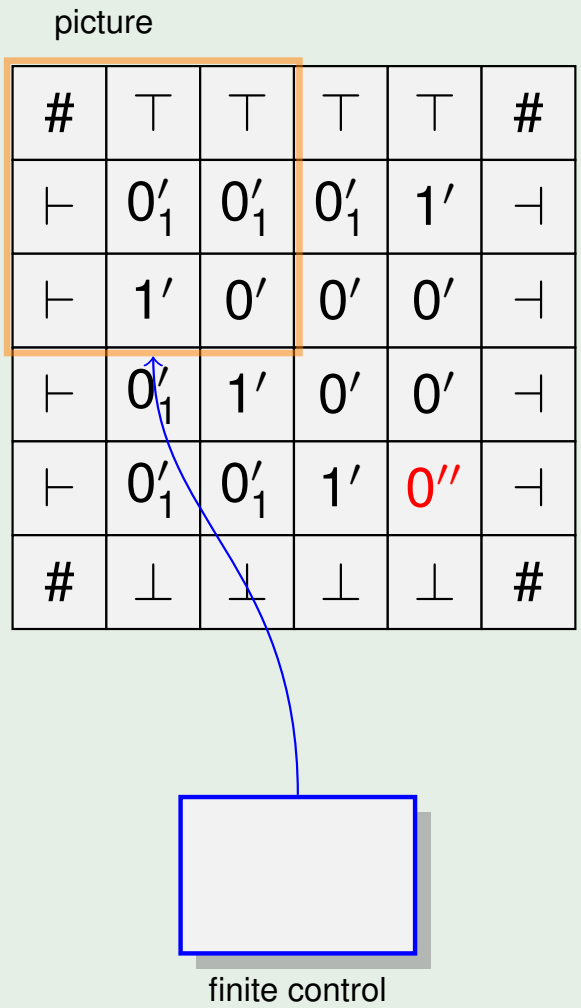

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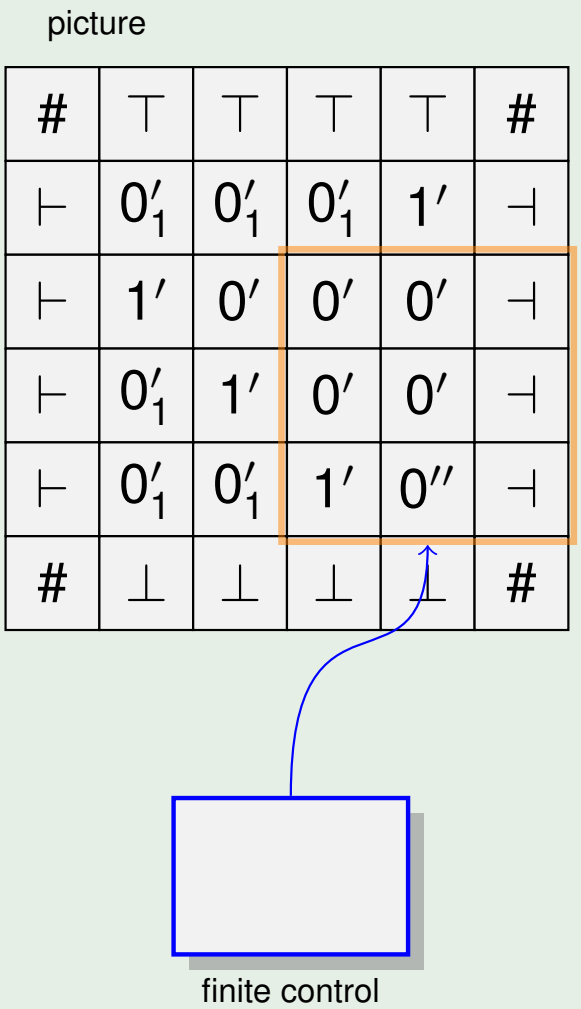

#	$\top$	$\top$	$\top$	$\top$	#
$\vdash$	$0'_1$	$0'_1$	$0'_1$	$1'$	$\dashv$
$\vdash$	$1'$	$0'$	$0'$	$0'$	$\dashv$
$\vdash$	$0'_1$	$1'$	$0'$	$0'$	$\dashv$
$\vdash$	$0'_1$	$0'_1$	$1'$	$0''$	$\dashv$
#	$\bot$	$\bot$	$\bot$	$\bot$	#

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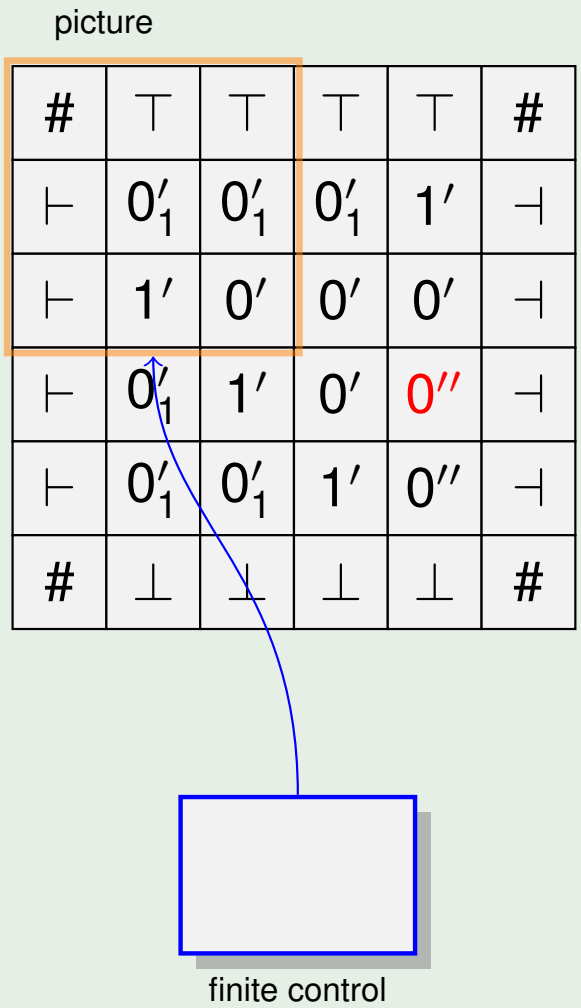

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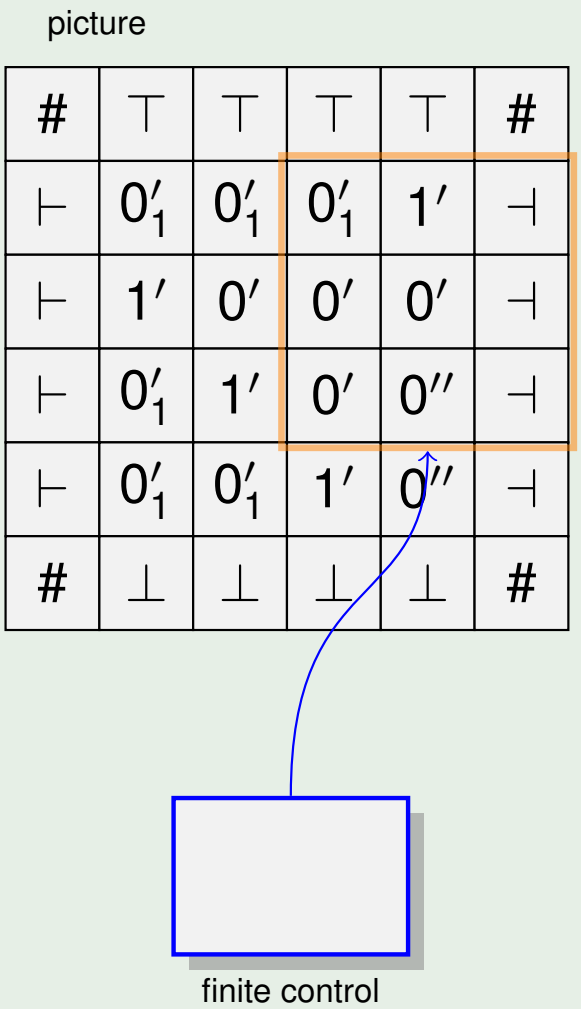

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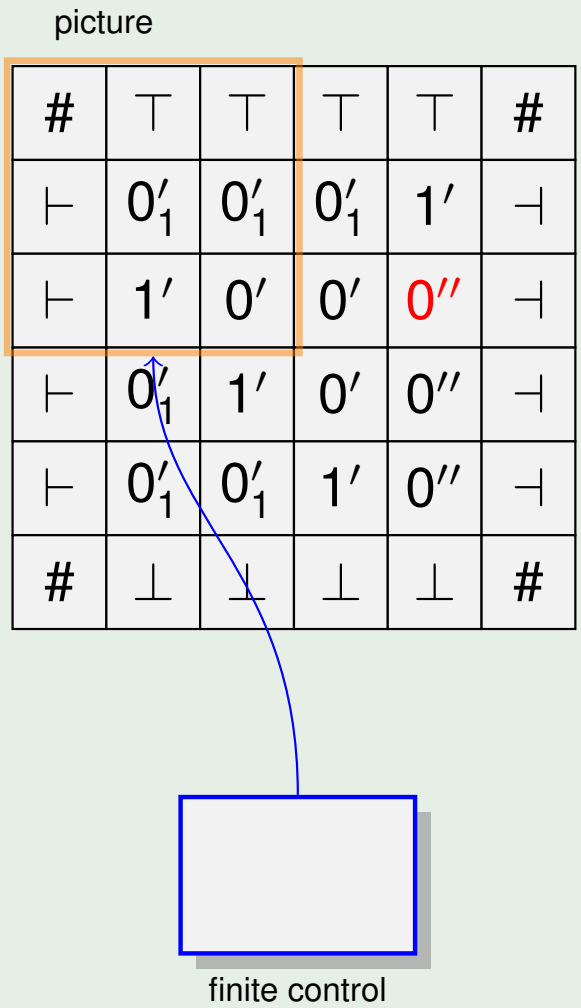

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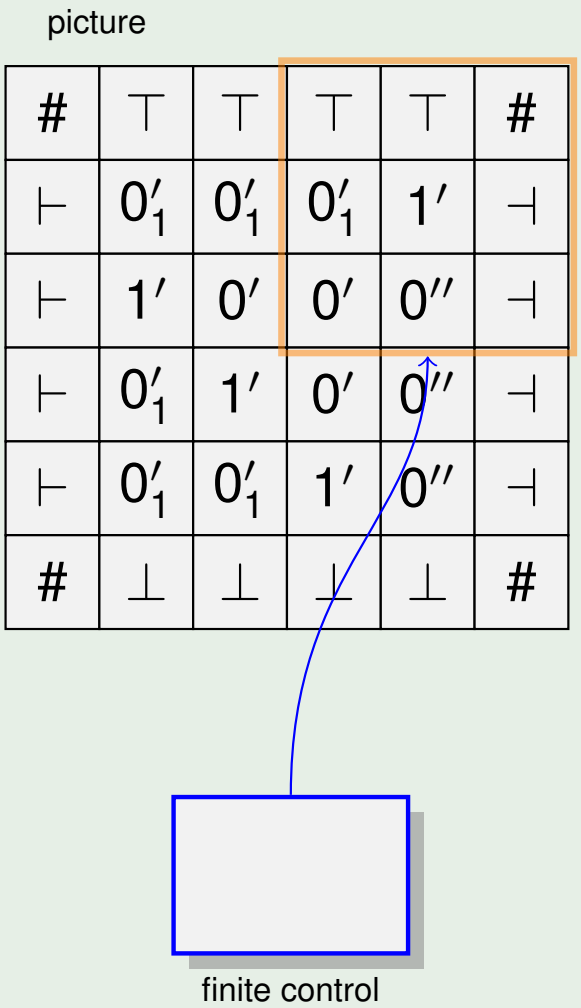

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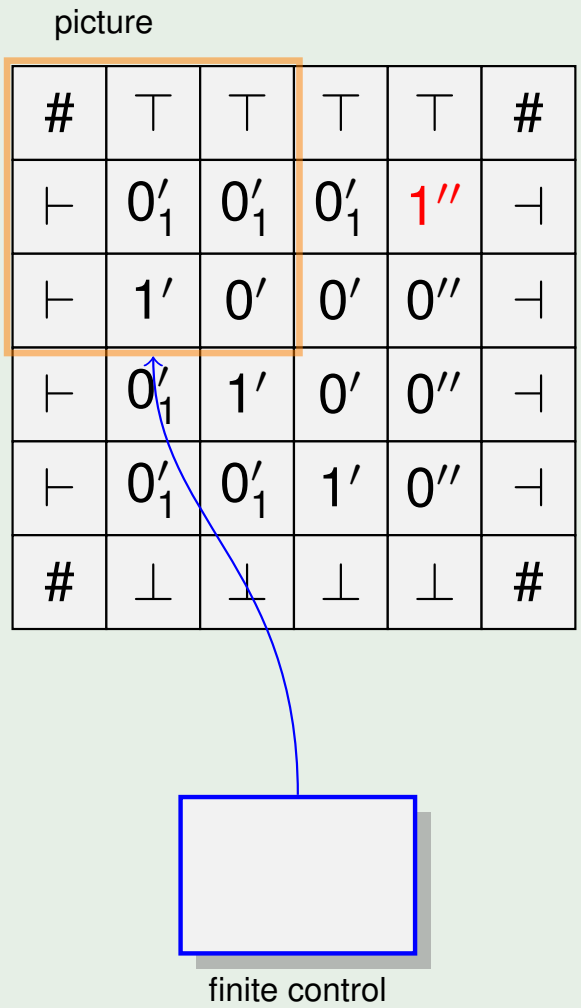
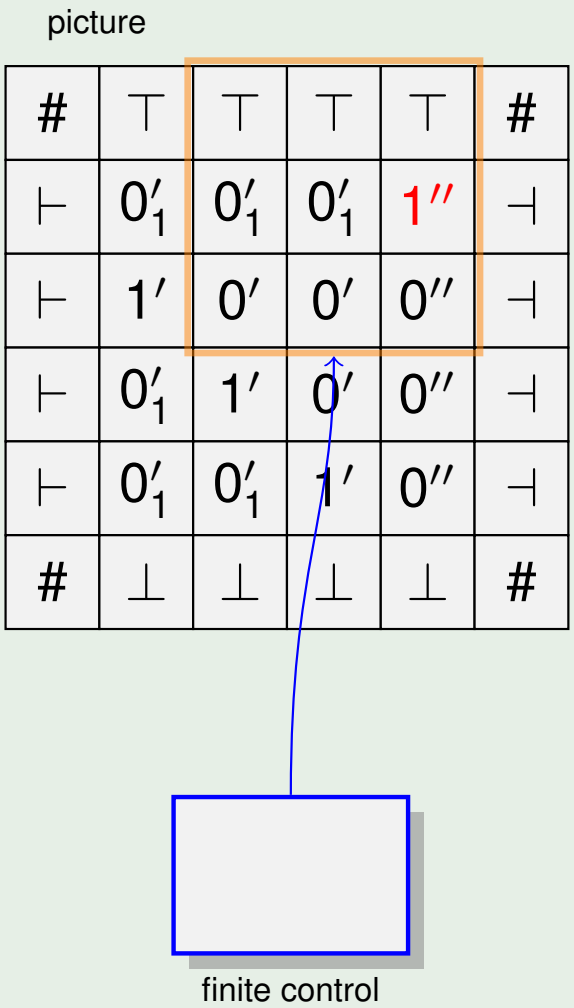
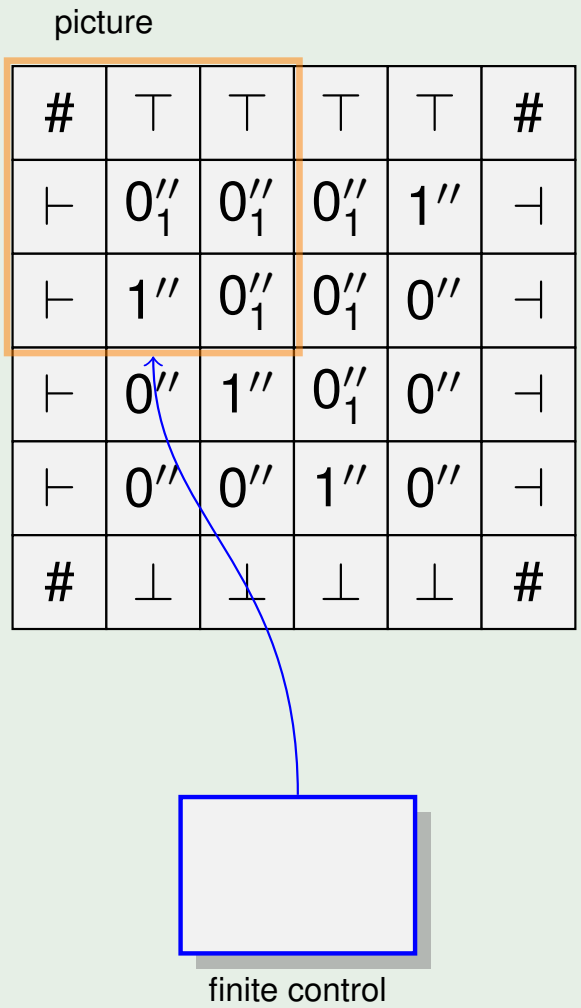


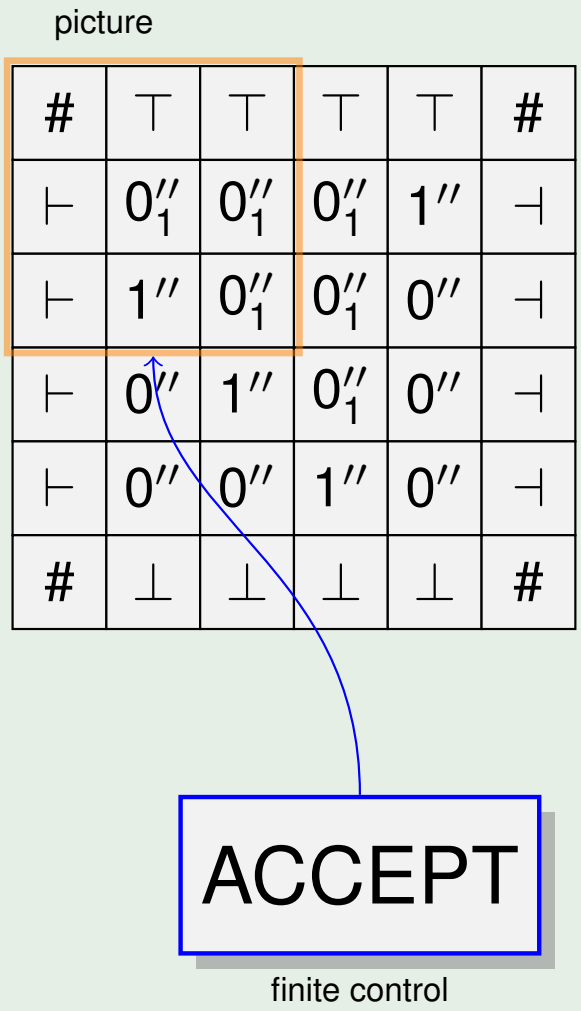
Example 1 (cont.)



Example 1 (cont.)



Example 1 (cont.)



4. On the Language Class $\mathcal{L}$(det-2D-x2W-ORWW)

Theorem 8

The stateless det-2D-x2W-ORWW-automaton can simulate the deterministic Sgraffito automaton.

Theorem 9

The classes of picture languages $\mathcal{L}$(det-2D-x2W-ORWW) *and* $\mathcal{L}$(stl-det-2D-x2W-ORWW) *are closed under transposition, union, intersection, and complementation.*

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In (M., O., SOFSEM 2014) it is shown that the language

$$L_{1\text{col}} = \{ P \in \Sigma^{2n,1} \mid P(1,1) \dots P(n,1) = (P(n+1,1) \dots P(2n,1))^R \},$$

is accepted by a det-2D-3W-ORWW-automaton.
The transpose $L_{1\text{col}}^t$ of this language is essentially the word language

$$L_{\text{pal}} = \{ w \in \{a,b\}^* \mid |w| \equiv 0 \bmod 2 \text{ and } w = w^R \} \notin \text{REG}.$$

Hence, $L_{1\text{col}}$ is not accepted by any det-2D-x2W-ORWW-automaton, i.e., there are det-2D-3W-ORWW-automata which cannot be simulated by det-2D-x2W-ORWW-automata.

However, also the converse holds.

Example 2

Let $L_{\mathrm{pal},2}$ be the following picture language over $\Sigma = \{a, b, \square\}$:

$$L_{\mathrm{pal},2} = \{\, P \in \Sigma^{2,2n} \mid \begin{array}{l} P(1,1)\dots P(1,n) = (P(1,n+1)\dots P(1,2n))^R, \\ P(1,i) \in \{a,b\} \text{ and } P(2,i) = \square,\ 1 \leq i \leq 2n \,\}, \end{array}$$

that is, $L_{\mathrm{pal},2}$ consists of all two-row pictures such that the first row contains a palindrome of even length over $\{a, b\}$, and the second row just contains $\square$-symbols.

We describe a det-2D-x2W-ORWW-automaton $M_{\mathrm{pal},2}$ for $L_{\mathrm{pal},2}$.

Let

$$\Gamma = \Sigma \cup \{a_1, a_2, b_1, b_2, \uparrow_1, \uparrow_2\},$$

and let

$$a > b > a_1 > b_1 > a_2 > b_2 > \square > \uparrow_1 > \uparrow_2 .$$

The automaton $M_{\mathrm{pal},2}$ proceeds as follows:

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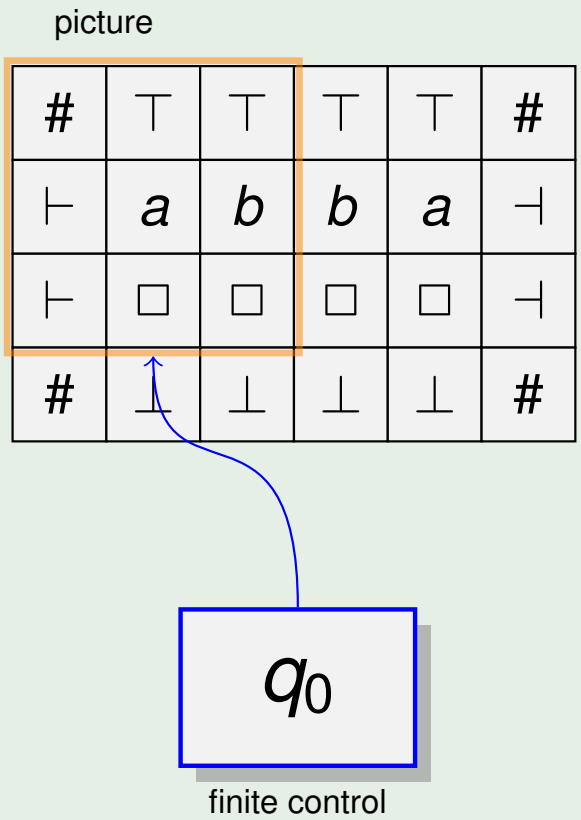
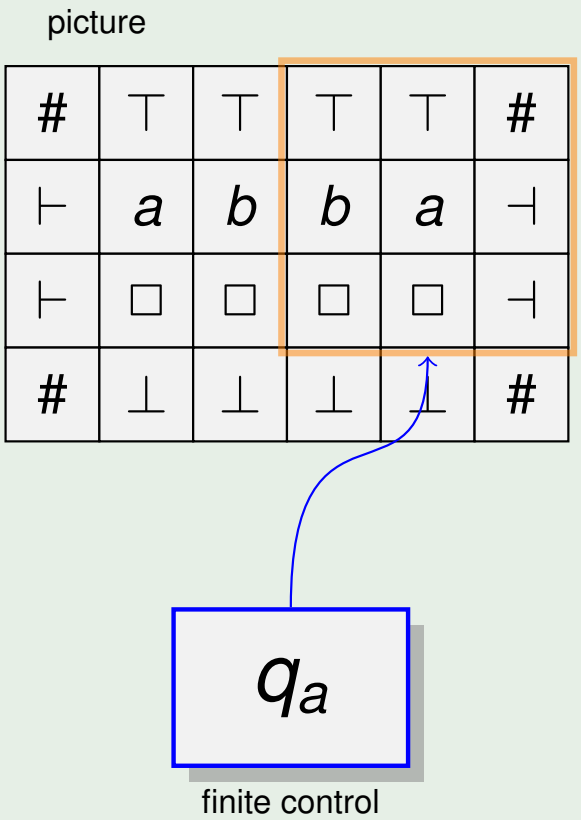
Example 2 (cont.)



Example 2 (cont.)



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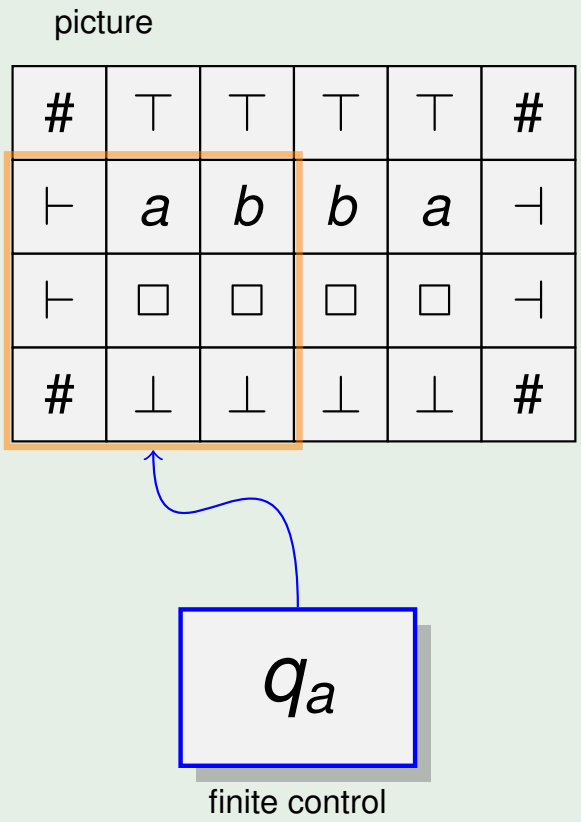

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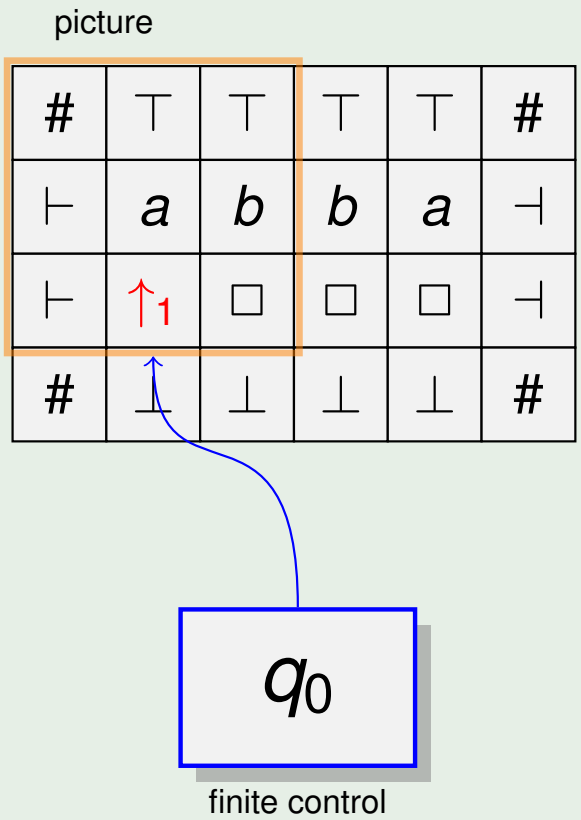
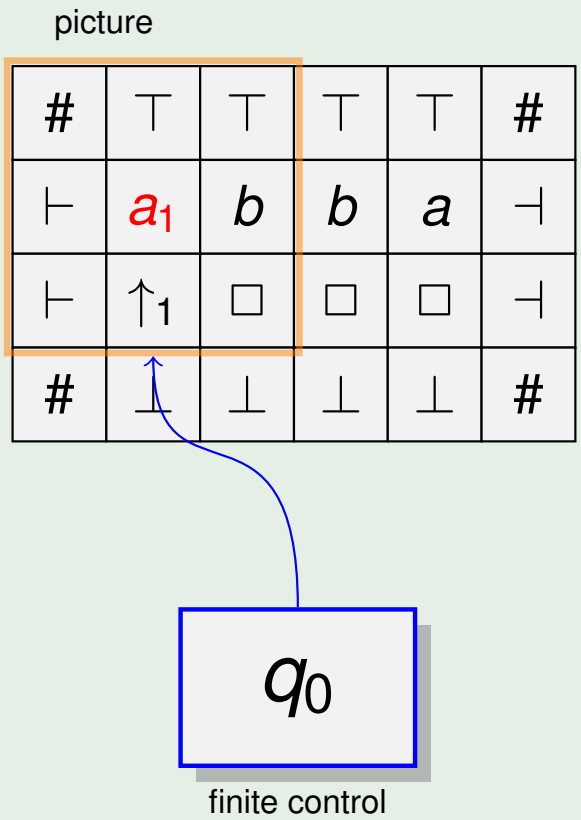
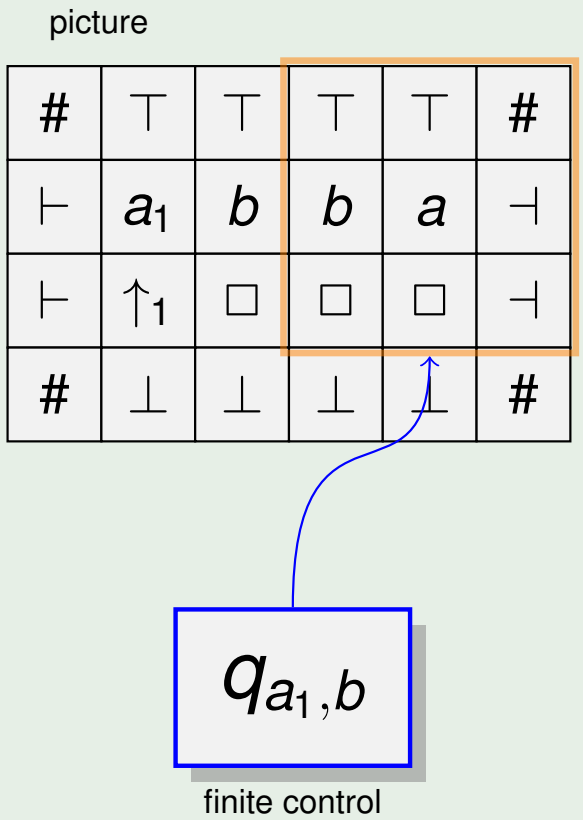


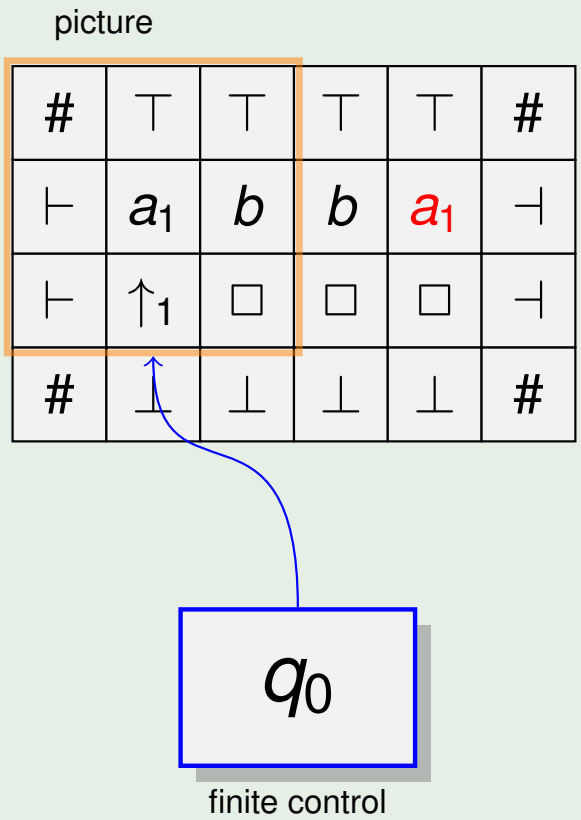
Example 2 (cont.)



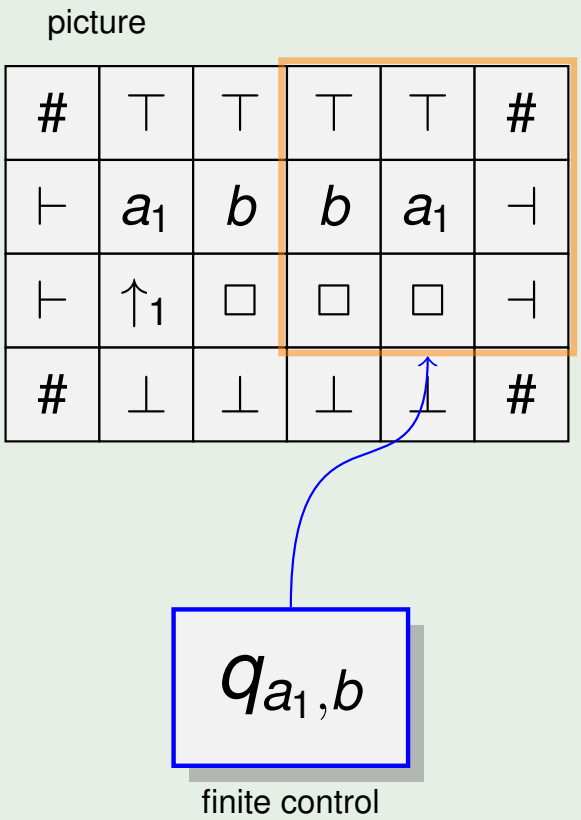
Example 2 (cont.)



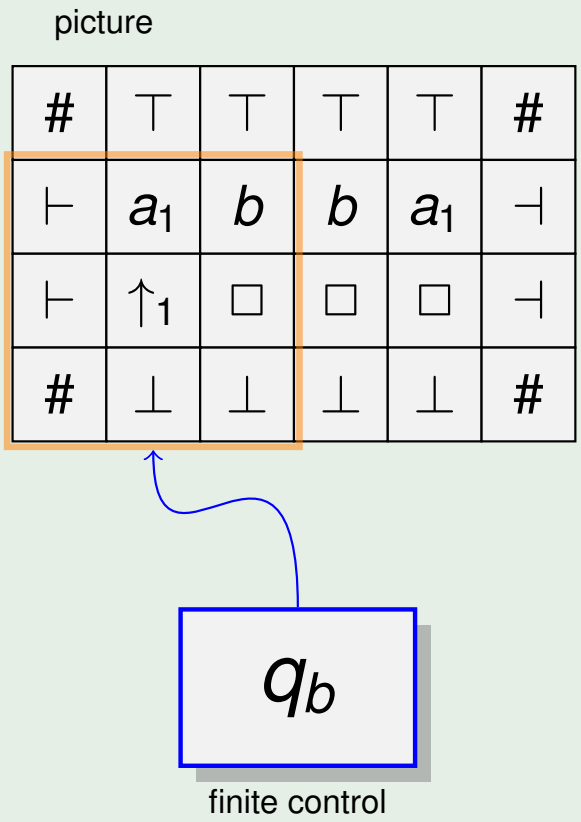
Example 2 (cont.)



Example 2 (cont.)



Example 2 (cont.)



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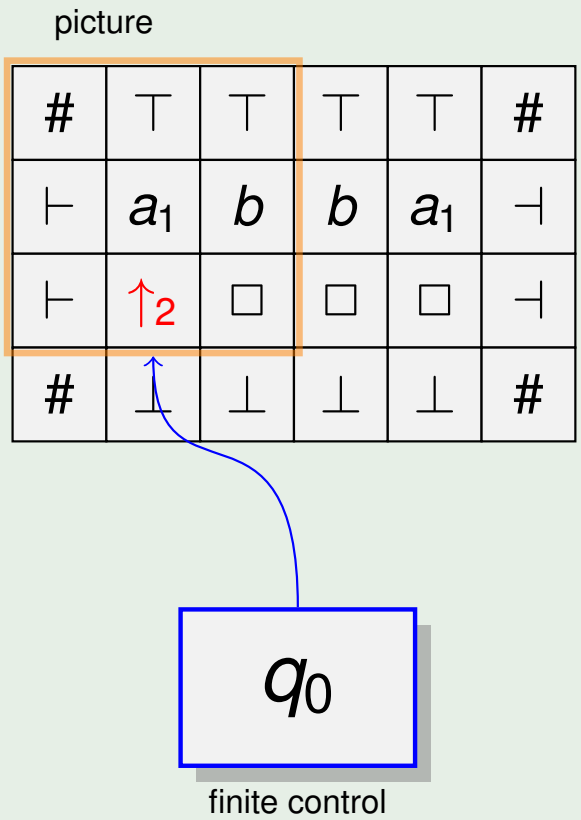
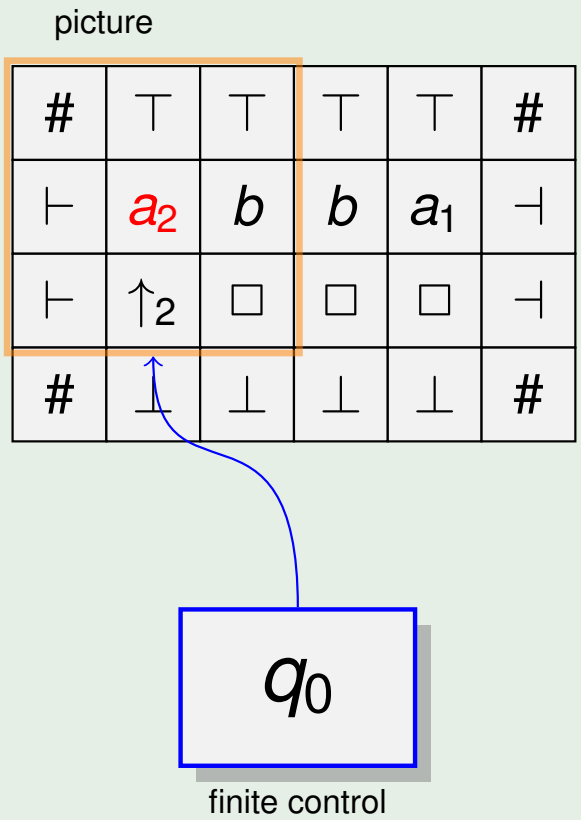


Example 2 (cont.)



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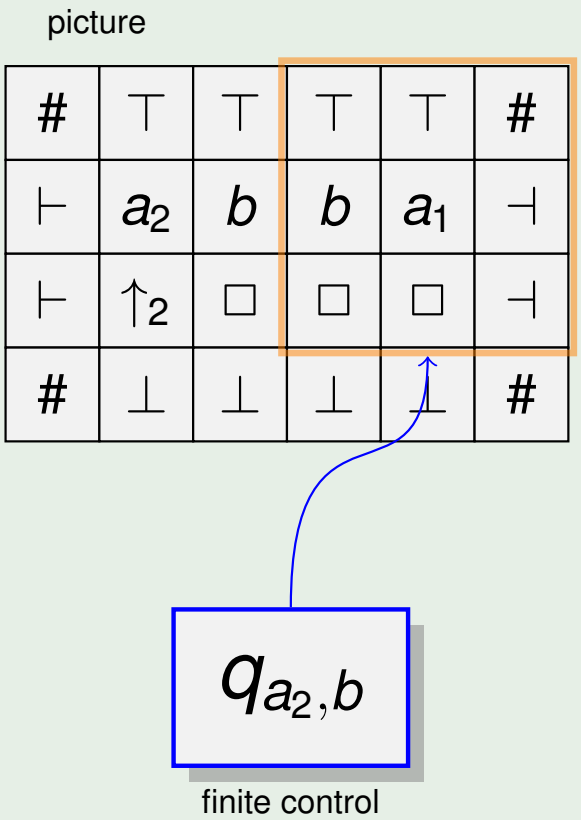

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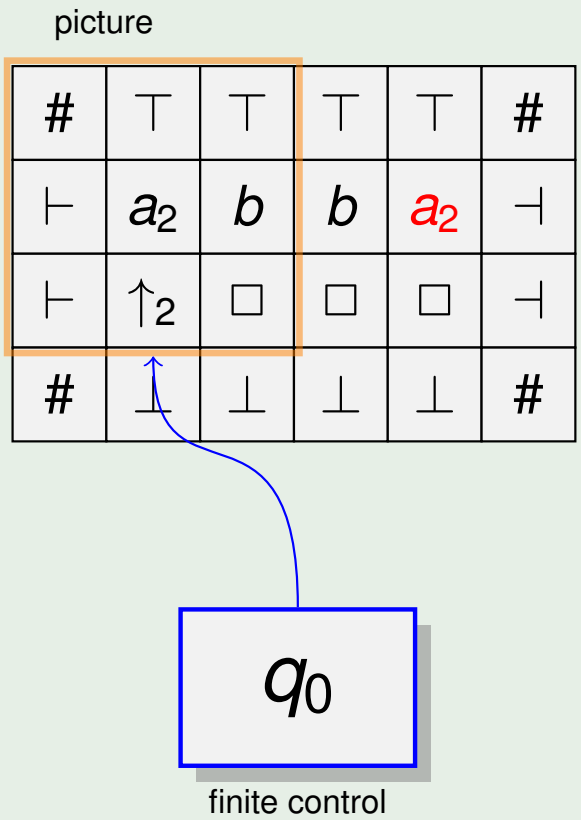

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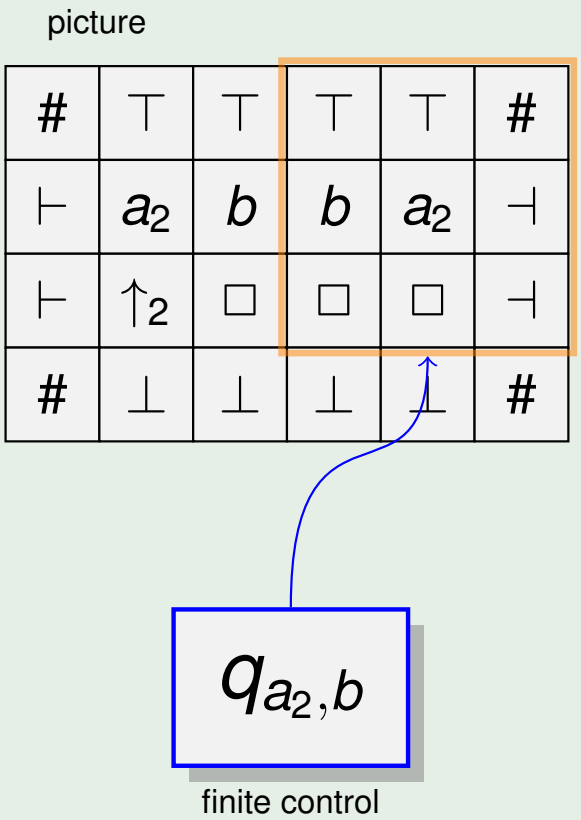

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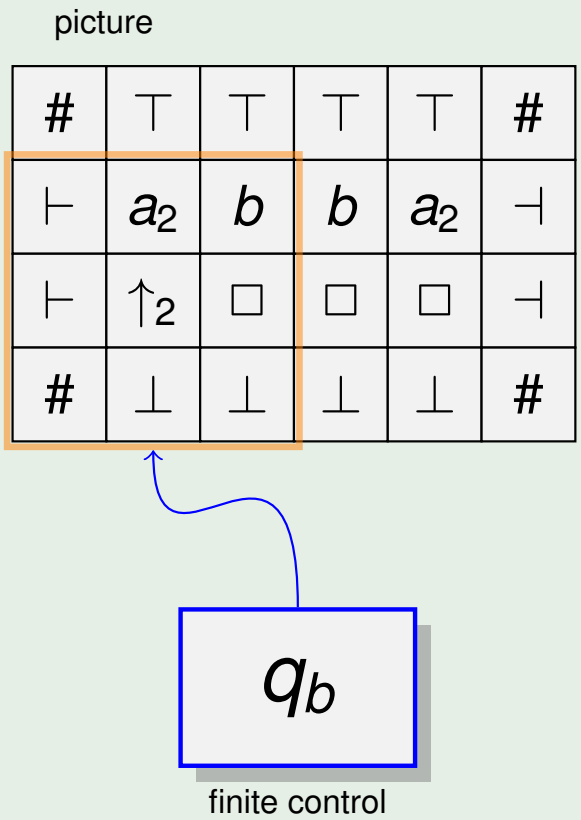

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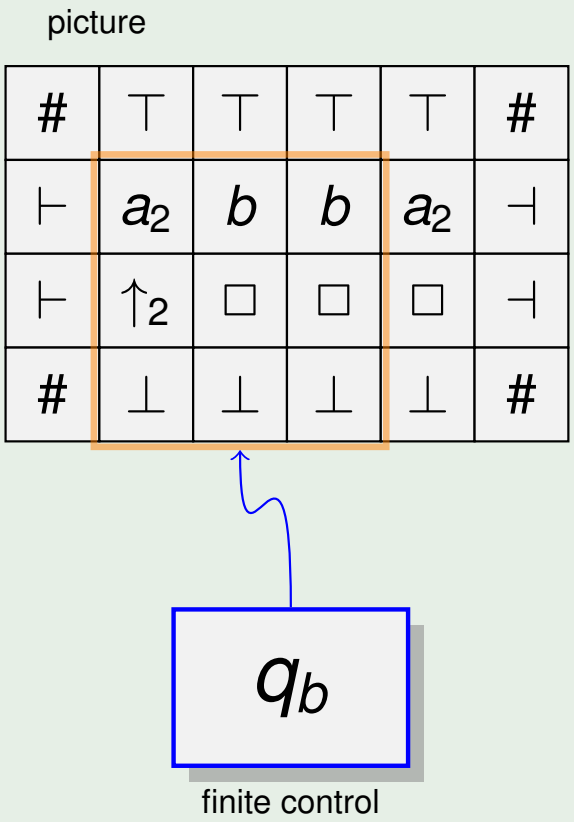

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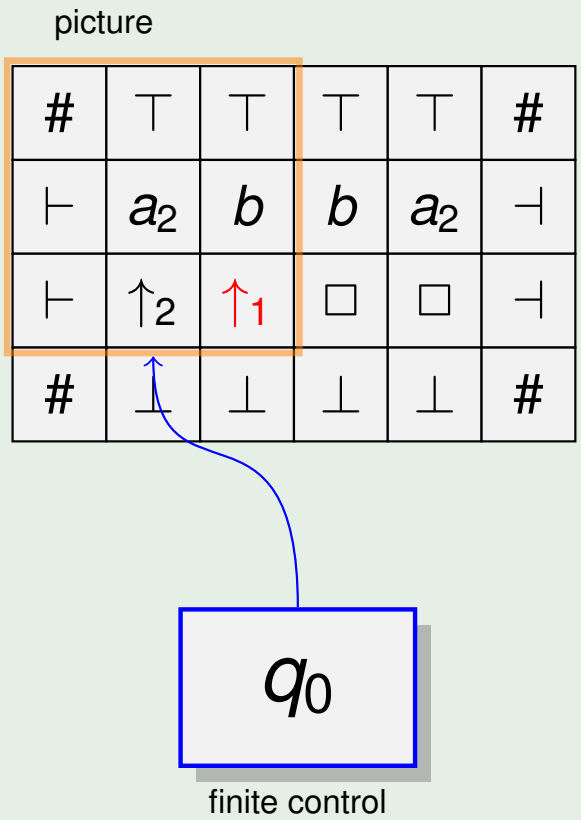

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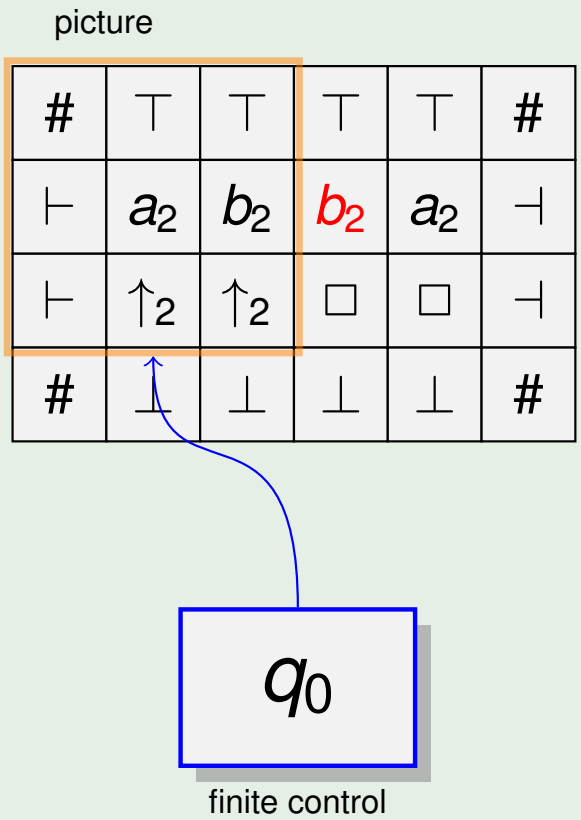

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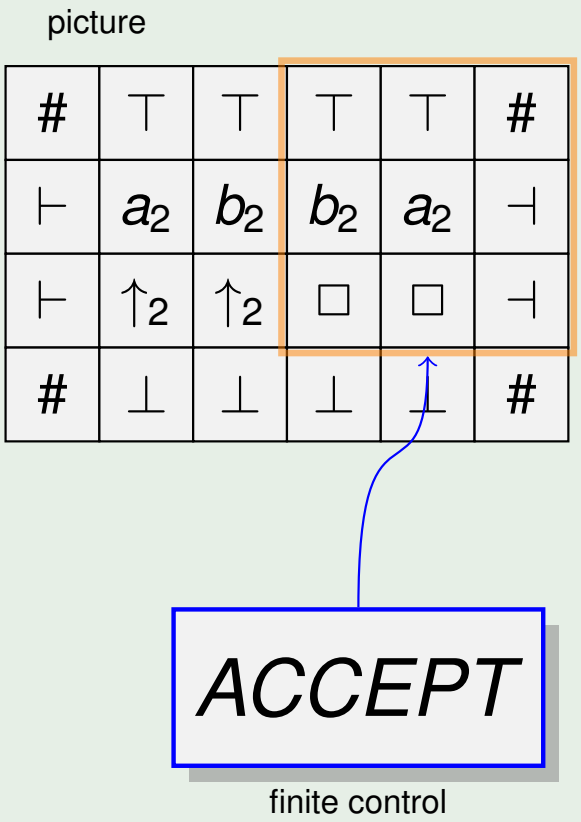

Proposition 10

$L_{\text{pal},2} \notin \mathcal{L}$(det-2D-3W-ORWW).

Corollary 11

The class of picture languages $\mathcal{L}$(det-2D-x2W-ORWW) *is incomparable under inclusion to the class of picture languages* $\mathcal{L}$(det-2D-3W-ORWW).

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$L_{\text{pal},2} \notin \mathcal{L}$(stl-det-2D-x2W-ORWW).

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$\mathcal{L}$(stl-det-2D-x2W-ORWW) $\subsetneq \mathcal{L}$(det-2D-x2W-ORWW).

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Proof outline of Prop. 12.

Assume that $M = (\Sigma, \Gamma, \mathcal{S}, \delta, >)$ is a stl-det-2D-x2W-ORWW-automaton over $\Sigma = \{a, b, \square\}$ such that $L(M) = L_{\text{pal},2}$.

For $w = a_1 \dots a_n$, where $n \geq 1$ and $a_1, \dots, a_n \in \{a, b\}$, let

$$P_w = \begin{bmatrix} a_1 & \dots & a_n & a & a & a_n & \dots & a_1 \\ \square & \dots & \square & \square & \square & \square & \dots & \square \end{bmatrix} \in L_{\text{pal},2}.$$

Given P_w as input, M will perform an accepting computation, which can be split into a finite number of phases, where we distinguish between four types of phases:

- A left-only phase consists of a sequence of cycles in which the window of M stays on the left half of the picture.
- An upper-right phase consists of a sequence of cycles in which all rewrite steps are performed on the right half of the picture, and in addition, in the first of these cycles, M enters the right half of the picture through a move-right step in row 1.

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Proof outline (cont.)

- A lower-left phase is a sequence of cycles in which all rewrite steps are performed in the left half of the picture, and in addition, the first of these cycles contains an extended move-right step.
- A lower-right phase is a sequence of cycles in which all rewrite steps are performed in the right half of the picture, and in addition, in the first of these cycles, M enters the right half of the picture through a move-right step in row 2 or through an extended move-down step.

The sequence of cycles of the computation of M on input P_w can uniquely be split into a sequence of phases of maximum length.

Thus, this computation can be described in a unique way by a string α over the alphabet $\Omega = \{O, U, L, R\}$, where O denotes a left-Only phase, U stands for an Upper-right phase, L denotes a lower-Left phase, and R stands for a lower-Right phase.

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Concerning the possible changes from one phase to the next there are some restrictions based on the fact that M is stateless, e.g.,

- while M is in a lower-right phase (R), it just moves through the left half of the current picture after each rewrite/restart step. Thus, M cannot get into another phase until it performs a rewrite step that replaces a symbol in the first column of the right half of the picture. However, in a fixed column, less than $2 \cdot |\Gamma|$ many rewrite steps can be performed, and so $|\alpha|_R \leq 1 + 2 \cdot |\Gamma|$.

It can be shown that

$$|\alpha| \leq |\alpha|_O + |\alpha|_R + |\alpha|_L + |\alpha|_U \leq 15 + 28 \cdot |\Gamma|,$$

i.e., each computation of M consists of $\leq 15 + 28 \cdot |\Gamma|$ many phases.

Using a notion of generalized crossing sequence and counting arguments it can now be shown that M will also accept some pictures that do not belong to the language $L_{\text{pal},2}$, a contradiction! □

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Let $\Sigma = \{0, 1\}$, and let

$$L_{\text{dup}} = \{\, P \oplus P \mid P \text{ is a quadratic picture over } \Sigma \,\},$$

where $P \oplus P$ denotes the column concatenation of two copies of P.

It is known that $L_{\text{dup}} \notin \mathcal{L}$(2SA) (Pruša, Mráz, DLT 2012) .

However, by using the technique from Example 2, the following can be shown.

Proposition 14

$L_{\text{dup}} \in \mathcal{L}$(det-2D-x2W-ORWW).

Corollary 15

$\mathcal{L}$(det-2D-x2W-ORWW) $\not\subset \mathcal{L}$(2SA).

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5. Concluding Remarks

The det-2D-x2W-ORWW-automaton has the following nice properties:

- The membership problem for the picture languages accepted is decidable in polynomial time.
- It is even more expressive than the deterministic Sgraffito automaton.
- It only accepts regular word languages.
- It avoids the artificial termination condition of the det-2D-3W-ORWW-automaton.

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Open Problems:

- Is the language class $\mathcal{L}$(det-2D-x2W-ORWW) closed under projection, under horizontal product, or under vertical product?
- Can stateless det-2D-x2W-ORWW-automata accept any languages that are not accepted by deterministic Sgraffito automata, or do these two types of automata have exactly the same expressive power?
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Thank you for your attention!