On the Diameter of Rearrangement Problems

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Carla Negri Lintzmayer
Zanoni Dias

University of Campinas (UNICAMP)
Institute of Computing
Campinas, São Paulo, Brazil

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2 Definitions

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- Find scenarios that show how to transform one genome into another
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- Prefix and suffix reversals and transpositions

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- Identity permutation: $\iota_n = (1\ 2\ \dots\ n)$

Definitions

- (Unsigned) reversal: $\rho(i, j)$ with $1 \leq i < j \leq n$

$$\begin{aligned}
\pi &= (\pi_1 \ \ldots \ \pi_{i-1} \ \underline{\pi_i \ \pi_{i+1} \ \ldots \ \pi_{j-1} \ \pi_j} \ \pi_{j+1} \ \ldots \ \pi_n) \\
\pi \cdot \rho(i,j) &= (\pi_1 \ \ldots \ \pi_{i-1} \ \underline{\pi_j \ \pi_{j-1} \ \ldots \ \pi_{i+1} \ \pi_i} \ \pi_{j+1} \ \ldots \ \pi_n)
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- Example:

$$\begin{aligned}\pi &= (3 \; \underline{1 \; 5 \; 2 \; 7} \; 4 \; 3) \\ \pi \cdot \rho(2, 5) &= (3 \; \underline{7 \; 2 \; 5 \; 1} \; 4 \; 3)\end{aligned}$$

Definitions

- Signed reversal: $\bar{\rho}(i, j)$ with $1 \leq i \leq j \leq n$

$$\begin{array}{rcl} \pi & = & (\pi_1 \ \dots \ \pi_{i-1} \ \underline{\pi_i \ \ \pi_{i+1} \ \dots \ \pi_{j-1} \ \ \pi_j} \ \pi_{j+1} \ \dots \ \pi_n) \\ \pi \cdot \bar{\rho}(i,j) & = & (\pi_1 \ \dots \ \pi_{i-1} \ \underline{-\pi_j \ -\pi_{j-1} \ \dots \ -\pi_{i+1} \ -\pi_i} \ \pi_{j+1} \ \dots \ \pi_n) \end{array}$$

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- Example:

$$\begin{array}{rl} \pi = & (-3 \;\underline{+1 \;\; -5 \;\; +2 \;\; +7}\; -4 \;\; -3) \\ \pi \cdot \bar{\rho}(2,5) = & (-3 \;\underline{-7 \;\; -2 \;\; +5 \;\; -1}\; -4 \;\; -3) \end{array}$$

Definitions

- Transposition: $\tau(i, j, k)$ with $1 \leq i < j < k \leq n+1$

$$\begin{array}{rcl}
\pi & = & (\pi_1 \ldots \pi_{i-1} \ \underline{\pi_i \ \pi_{i+1} \ldots \pi_{j-1}} \ \underline{\pi_j \ \pi_{j+1} \ldots \pi_{k-1}} \ \pi_k \ldots \pi_n) \\
\pi \cdot \tau(i,j,k) & = & (\pi_1 \ldots \pi_{i-1} \ \underline{\pi_j \ \pi_{j+1} \ldots \pi_{k-1}} \ \underline{\pi_i \ \pi_{i+1} \ldots \pi_{j-1}} \ \pi_k \ldots \pi_n)
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Definitions

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$$\begin{array}{rcl} \pi & = & (\pi_1 \dots \pi_{i-1}\ \underline{\pi_i\ \ \pi_{i+1} \dots \pi_{j-1}}\ \underline{\pi_j\ \ \pi_{j+1} \dots \pi_{k-1}}\ \pi_k \dots \pi_n) \\ \pi{\cdot}\tau(i{,}j{,}k) & = & (\pi_1 \dots \pi_{i-1}\ \underline{\pi_j\ \ \pi_{j+1} \dots \pi_{k-1}}\ \underline{\pi_i\ \ \pi_{i+1} \dots \pi_{j-1}}\ \pi_k \dots \pi_n) \end{array}$$

- Example:

$$\begin{array}{rl} \pi = & (3\ \underline{1\ 5}\ \underline{2\ 7\ 4}\ 3) \\ \pi \cdot \tau(2,4,7) = & (3\ \underline{2\ 7\ 4}\ \underline{1\ 5}\ 3) \end{array}$$

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Definitions

- Prefix reversal (inverts first segment):
  - unsigned: $\rho_p(j) \equiv \rho(1, j)$ for $1 < j \leq n$
  - signed: $\bar{\rho}_p(j) \equiv \bar{\rho}(1, j)$ for $1 \leq j \leq n$

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- Suffix reversal (inverts last segment):
  - unsigned: $\rho_s(i) \equiv \rho(i, n)$ for $1 \leq i < n$
  - signed: $\bar{\rho}_s(i) \equiv \bar{\rho}(i, n)$ for $1 \leq i \leq n$

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- Prefix transposition: $\tau_p(j, k) \equiv \tau(1, j, k)$ for $1 < j < k \leq n + 1$
- Suffix transposition: $\tau_s(i, j) \equiv \tau(i, j, n+1)$ for $1 \leq i < j < n + 1$

Definitions

- Rearrangement model: β

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- Distance: $d_\beta(\pi)$, minimum number of operations in β needed to transform π into ι_n

Definitions

Diameter: $D_\beta(n)$

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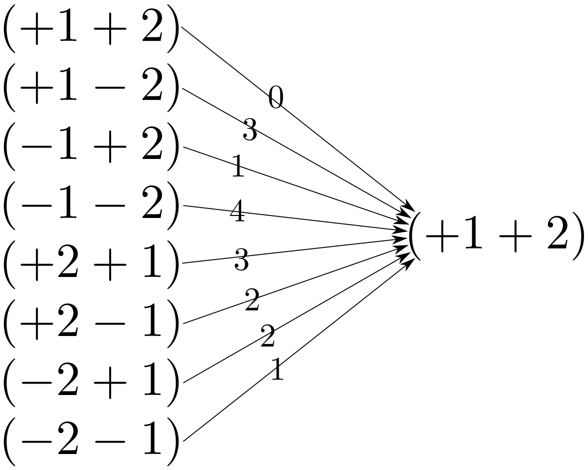

$$D_{\bar{\rho_p}}(2) = 4$$

Definitions

- Breakpoint: occurs between two consecutive elements of π that should not be consecutive

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- Transpositions:

$$(0 \;\blacksquare\; 3 \blacksquare 2 \blacksquare 4\;5 \blacksquare 1 \;\blacksquare\; 6)$$

- Signed:

$$(+0 \;\blacksquare\; -3 \;-2 \blacksquare -4 \blacksquare -5 \blacksquare +1 \;\blacksquare\; +6)$$

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Problems

Problem	Diameter
SbR	$D_\rho(n) = n - 1$ [1]
SbSigR	$D_{\bar{\rho}}(n) = n + 1$ [2]
SbT	$\left\lfloor \frac{n+1}{2} \right\rfloor \leq D_\tau(n) \leq \left\lfloor \frac{2n-2}{3} \right\rfloor$ [3, 4]
SbRT	?
SbSigRT	$\left\lfloor \frac{n}{2} \right\rfloor + 2 \leq D_{\bar{\rho}\tau}(n)$ [5]
SbPR	$\frac{15n}{14} \leq D_{\rho_p}(n) \leq \frac{18n}{11} + O(1)$ [6, 7]
SbSigPR	$\frac{3n+3}{2} \leq D_{\bar{\rho_p}}(n) \leq 2n - 6$ [6, 8]
SbPT	$\left\lfloor \frac{3n+1}{4} \right\rfloor \leq D_{\tau_p}(n) \leq n - \log_{\frac{9}{2}} n$ [9, 10]
SbPRPT	?

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Results

- We found families of permutations for 8 problems:
  - SbRT
  - SbPRPT
  - SbSigPRPT
  - SbPRSR
  - SbSigPRSigSR
  - SbPTST
  - SbPRPTSRST
  - SbSigPRPTSigSRST
- Lower and upper bounds on the diameters

Results

Family for SbPRSR:

$$\pi_n^* = \begin{cases} (n\ \ 1\ \ n{-}2\ \ n{-}4\ \ n{-}6\ \ ...\ \ 4\ \ 2\ \ n{-}3\ \ n{-}5\ \ n{-}7\ \ ...\ \ 3\ \ n{-}1) & \text{if } n \text{ is even} \\ (n\ \ 1\ \ n{-}2\ \ n{-}4\ \ n{-}6\ \ ...\ \ 5\ \ 3\ \ n{-}3\ \ n{-}5\ \ n{-}7\ \ ...\ \ 2\ \ n{-}1) & \text{if } n \text{ is odd} \end{cases}$$

Results

Family for SbPRSR:

$$\pi_n^* = \begin{cases} (n \;\; 1 \;\; n\text{–}2 \;\; n\text{–}4 \;\; n\text{–}6 \;\; \dots \;\; 4 \;\; 2 \;\; n\text{–}3 \;\; n\text{–}5 \;\; n\text{–}7 \;\; \dots \;\; 3 \;\; n\text{–}1) & \text{if } n \text{ is even} \\ (n \;\; 1 \;\; n\text{–}2 \;\; n\text{–}4 \;\; n\text{–}6 \;\; \dots \;\; 5 \;\; 3 \;\; n\text{–}3 \;\; n\text{–}5 \;\; n\text{–}7 \;\; \dots \;\; 2 \;\; n\text{–}1) & \text{if } n \text{ is odd} \end{cases}$$

$$\begin{aligned} n = 9 &: \;\; (9\ 1\ 7\ 5\ 3\ 6\ 4\ 2\ 8) \\ n = 10 &: \;\; (10\ 1\ 8\ 6\ 4\ 2\ 7\ 5\ 3\ 9) \\ n = 11 &: \;\; (11\ 1\ 9\ 7\ 5\ 3\ 8\ 6\ 4\ 2\ 10) \\ n = 12 &: \;\; (12\ 1\ 10\ 8\ 6\ 4\ 2\ 9\ 7\ 5\ 3\ 11) \end{aligned}$$

Results

Lemma

For $n \geq 8$, $n - 1 \leq d_{\rho_p \rho_s}(\pi_n^*) \leq n$.

Proof.

The lower bound is true because $b_{\rho_p \rho_s}(\pi_n^*) = n - 1$ and $d_{\rho_p \rho_s}(\pi) \geq b_{\rho_p \rho_s}(\pi)$ for any π [11]. The upper bound is true because of next algorithm. □

Results

Algorithm to sort π_n^*:

Input: $\pi = \pi_n^*$, $n \geq 8$

$\pi \leftarrow \pi \cdot \rho_p(n-1)$

$\pi \leftarrow \pi \cdot \rho_p(n-3)$

$\pi \leftarrow \pi \cdot \rho_s(2)$

$\pi \leftarrow \pi \cdot \rho_s(\pi^{-1}_{\pi_n+1} + 1)$

$\pi \leftarrow \pi \cdot \rho_p(\pi^{-1}_{\pi_1-1} - 1)$

$\pi \leftarrow \pi \cdot \rho_s(\pi^{-1}_n)$

while $\pi_1 \neq 1$ **do**

$\quad \pi \leftarrow \pi \cdot \rho_p(\pi^{-1}_{\pi_1+1} - 1)$

Results

Lemma

For $n \geq 8$, $D_{\rho_p \rho_s}(n) \geq n - 1$ *and for* $n \geq 1$, $D_{\rho_p \rho_s}(n) \leq \frac{18n}{11} + O(1)$.

Proof.

The lower bound is true because of family π_n^*. The upper bound is true because $D_{\rho_p \rho_s}(n) \leq D_{\rho_p}(n)$, since $d_{\rho_p \rho_s}(\pi) \leq d_{\rho_p}(\pi)$ for any π. This is true because any sorting sequence for Sorting by Prefix Reversals is valid for Sorting by Prefix Reversals and Suffix Reversals. □

Results

- We know that $D_{\rho_p \rho_s}(n) = n$ for $7 \leq n \leq 13$
- We verified that $d_{\rho_p \rho_s}(\pi_n^*) = n$ for $8 \leq n \leq 15$

Results

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- We verified that $d_{\rho_p\rho_s}(\pi_n^*) = n$ for $8 \leq n \leq 15$

Conjecture

For $n \geq 8$, $D_{\rho_p\rho_s}(n) = d_{\rho_p\rho_s}(\pi_n^*) = n$.

Results

Problem	Diameter
SbRT	$\left\lceil \frac{n}{2} \right\rceil \leq D_{\rho\tau}(n) \leq \left\lfloor \frac{2n-2}{3} \right\rfloor$
SbPRPT	$\left\lceil \frac{n}{2} \right\rceil \leq D_{\rho_p\tau_p}(n) \leq n - \log_{\frac{9}{2}} n$
SbSigPRPT	$\left\lceil \frac{n}{2} \right\rceil + 1 \leq D_{\bar{\rho}_p\tau_p}(n) \leq \frac{18n}{11} + O(1)$
SbPRSR	$n - 1 \leq D_{\rho_p\rho_s}(n) \leq \frac{18n}{11} + O(1)$
SbSigPRSigSR	$n \leq D_{\bar{\rho}_p\bar{\rho}_s}(n) \leq 2n - 6$
SbPTST	$\left\lceil \frac{n-1}{2} \right\rceil + 1 \leq D_{\tau_p\tau_s}(n) \leq n - \log_{\frac{9}{2}} n$
SbPRPTSRST	$\left\lceil \frac{n}{2} \right\rceil \leq D_{\rho_p\tau_p\rho_s\tau_s}(n) \leq n - \log_{\frac{9}{2}} n$
SbSigPRPTSigSRST	$\left\lceil \frac{n-1}{2} \right\rceil \leq D_{\bar{\rho}_p\tau_p\bar{\rho}_s\tau_s}(n) \leq n + 1$

Results

We know that

1. $D_{\rho\tau}(n) = \left\lceil \frac{n}{2} \right\rceil$ for $4 \leq n \leq 13$
2. $D_{\rho_p \rho_s}(n) = n$ for $7 \leq n \leq 13$
3. $D_{\bar{\rho}_p \bar{\rho}_s}(n) = n + \left\lfloor \frac{n-1}{2} \right\rfloor$ for $5 \leq n \leq 10$
4. $D_{\rho_p \tau_p \rho_s \tau_s}(n) = \left\lceil \frac{n}{2} \right\rceil + 1$ for $6 \leq n \leq 13$

Results

It is also possible to validate that

1. $d_{\rho_p \rho_s}(\pi_n) = n$ for $8 \leq n \leq 15$
2. $d_{\bar{\rho}_p \bar{\rho}_s}(\pi_n) = n + \left\lfloor \frac{n-1}{2} \right\rfloor$ for $5 \leq n \leq 12$
3. $d_{\rho_p \tau_p \rho_s \tau_s}(\pi_n) = \left\lceil \frac{n}{2} \right\rceil + 1$ for $6 \leq n \leq 15$
4. $D_{\bar{\rho}_p \tau_p}(n) = d_{\bar{\rho}_p \tau_p}(\pi_n)$ for $2 \leq n \leq 10$
5. $D_{\tau_p \tau_s}(n) = d_{\tau_p \tau_s}(\pi_n)$ for $1 \leq n \leq 12$
6. $D_{\bar{\rho}_p \tau_p \bar{\rho}_s \tau_s}(n) = d_{\bar{\rho}_p \tau_p \bar{\rho}_s \tau_s}(\pi_n)$ for $n \in \{8, 10\}$ and $D_{\bar{\rho}_p \tau_p \bar{\rho}_s \tau_s}(n) = d_{\bar{\rho}_p \tau_p \bar{\rho}_s \tau_s}(\pi_n)$ for $n \in \{7, 9\}$

Results

Conjecture

For $n \geq 4$, $D_{\rho\tau}(n) = d_{\rho\tau}(\pi_n) = \left\lceil \frac{n}{2} \right\rceil$.

Conjecture

For $n \geq 8$, $D_{\rho_p\rho_s}(n) = d_{\rho_p\rho_s}(\pi_n) = n$.

Conjecture

For $n \geq 5$, $D_{\bar{\rho}_p\bar{\rho}_s}(n) = d_{\bar{\rho}_p\bar{\rho}_s}(\pi_n) = n + \left\lfloor \frac{n-1}{2} \right\rfloor$.

Conjecture

For $n \geq 6$, $D_{\rho_p\tau_p\rho_s\tau_s}(n) = d_{\rho_p\tau_p\rho_s\tau_s}(\pi_n) = \left\lceil \frac{n}{2} \right\rceil + 1$.

Results

Conjecture

For $n \geq 2$, $D_{\bar{\rho}_p \tau_p}(n) = d_{\bar{\rho}_p \tau_p}(\pi_n)$.

Conjecture

For $n \geq 1$, $D_{\tau_p \tau_s}(n) = d_{\tau_p \tau_s}(\eta_n) = n - \left\lfloor \frac{n}{3} \right\rfloor$.

Conjecture

For $n \geq 8$ *and* n *even*, $D_{\bar{\rho}_p \tau_p \bar{\rho}_s \tau_s}(n) = d_{\bar{\rho}_p \tau_p \bar{\rho}_s \tau_s}(\pi_n)$. *For* $n \geq 7$ *and* n *odd*, $D_{\bar{\rho}_p \tau_p \bar{\rho}_s \tau_s}(n) = d_{\bar{\rho}_p \tau_p \bar{\rho}_s \tau_s}(\pi_n)$.

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- We verified that the conjecture of Meidanis *et al.* [12] that $D_{\bar{\rho}\tau}(n) = \left\lfloor \frac{n}{2} \right\rfloor + 2$ is not true
- It is also not true that $D_{\bar{\rho}\tau}(n) = d_{\bar{\rho}\tau}(\bar{\iota_n})$

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- We verified that the conjecture of Meidanis *et al.* [12] that $D_{\bar{\rho}\tau}(n) = \left\lfloor \frac{n}{2} \right\rfloor + 2$ is not true
- It is also not true that $D_{\bar{\rho}\tau}(n) = d_{\bar{\rho}\tau}(\bar{\iota_n})$
- We keep working on these problems trying to improve the results

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Thank you!

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